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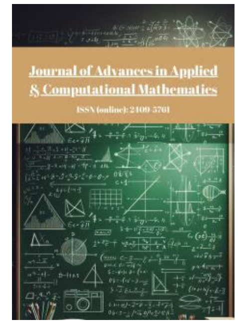

Dynamic Event-triggered H_∞ State Estimation for Memristive Neural Networks with Variance Constraints and Time-delay: A Finite-horizon Approach

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ABSTRACT

This paper discusses the dynamic event-triggered H_∞ state estimation issue for memristive neural networks with time-delay under variance constraints. The dynamic event-triggered mechanism is incorporated into the sensor-to-estimator to reduce resource consumption in the communication channel. The objective is to design the time-varying state estimator such that, in the presence of the dynamic event-triggered mechanism and time-delay, new sufficient criteria are derived to ensure the desired H_∞ performance and the boundedness of estimation error variance. Furthermore, a novel non-augmented H_∞ state estimation algorithm is proposed under variance constraint by using the stochastic analysis techniques. Finally, a simulation example is used to illustrate the effectiveness of the proposed H_∞ state estimation algorithm.

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1. Introduction

Over the past years, due to the important applications of neural networks (NNs) in practical systems such as optimization problems, pattern recognition and associative memory [1-3], many scholars have begun to pay attention to the research on related problems of NNs. With the deepening of research, the concept of memristor was proposed in [4], and researchers from Hewlett-Packard have also confirmed the existence of memristor in [5]. Memristor is the fourth new passive nano-information device following three basic circuit components of resistance, capacitance and inductance [6, 7]. Different from the existing devices, the memristor has the advantages of low energy consumption, non-volatility, small size and so on [8-10]. Actually, the memristor is very similar to biological synapses in both structure and function. Thus, more and more researchers use memristors to replace synapses in artificial NNs. Among the existing research results, the state estimation (SE) issue has attracted much attention and become an important research topic of memristive NNs (MNNs) [11-14]. Generally, the state of neurons is not completely measurable, we need to present an appropriate SE method to estimate the state of neurons [15-18]. For example, a new finite-horizon H_∞ SE scheme has been proposed in [19] for MNNs, where both the time-delay and stochastic communication protocol have been taken into consideration, and a sufficient condition has been given to ensure the H_∞ performance index. However, it is worth noting that the research on the H_∞ SE problem for MNNs remains limited and thus deserves further investigation.

Generally speaking, due to the fact that the circuit implementation of large-scale MNNs often consumes substantial resources, the problem of resource saving has become a hot topic for MNNs [20-22]. It is noteworthy that the dynamic event-triggered mechanism (DETM) can effectively save resources [23], which has a strong practical background, but unfortunately, it has received limited attention due to its mathematical complexity. In addition, most event-triggered mechanisms are static, that is, the threshold of the triggering condition is fixed (not adaptive or dynamic) [24, 25]. Up to now, there are few results regarding the SE problem of MNNs under DETM [26, 27]. Different from the static event-triggered mechanism, the DETM can reduce the frequency of event triggering, so as to avoid unnecessary data transmission and achieve satisfactory performance. For instance, in [27], the SE issue has been solved for delayed MNNs under DETM, where a sufficient condition has been given to ensure the desired H_∞ performance requirement. Note that there are relatively few results regarding the variance-constrained H_∞ SE problem for MNNs under DETM. In [28], the DETM has been adopted and the recursive distributed filtering algorithm has been proposed for discrete nonlinear systems. Based on the DETM in [28], this paper integrates the DETM into the multi-index framework to investigate the SE problem for MNNs. Consequently, in contrast to existing methods, sufficient criteria are derived to guarantee the desired H_∞ performance and the error variance boundedness (EVB), and the multi-index SE algorithm is further proposed for MNNs from wider application viewpoint. As such, how to utilize DETM to coordinate massive data transmission between MNNs and a remote estimator has important practical significance, which is also one of the motivations of our research.

Time-delay commonly occurs when signals are transmitted between neurons, mainly due to the limited communication time between neurons and the switching speed of amplifiers. In the networked environments, different types of time-delay issues have attracted increasing research attention because the existence of time-delays leads to undesired oscillations or even instability [29-31]. Specifically, in [32], a new event-based extended dissipative SE method has been proposed for memristor-based Markovian NNs with time-varying delays. In [33], the H_∞ SE problem has been addressed for NNs with mixed time-varying delays, and a sufficient condition has been derived to guarantee the desired H_∞ performance requirement [34, 35]. It should be noted that the time-delay effect may degrade the estimation performance. Recently, an H_∞ SE method has been proposed in [36] for recurrent NNs with time-varying delays, where sufficient conditions have been derived to guarantee the H_∞ performance index. In addition, the SE method under variance constraints is capable of offering a more relaxed technical approach that can characterize the allowable accuracy of the proposed H_∞ state estimation algorithm. Up to now, novel H_∞ SE algorithms have been presented in [37] and [38] for time-varying systems under variance constraint. Motivated by the above results, we attempt to address the H_∞ variance-constrained SE problem for MNNs with time-delay under DETM.

Summarizing the above discussions, the main aim is to propose a new H_∞ SE algorithm for MNNs subject to time-delay under DETM, which can guarantee two requirements including the H_∞ performance index and the EVB. The key technical challenges we tackled are as follows: i) How to propose appropriate method to handle the effects of activation function? ii) How to ensure the satisfactory estimation performance by utilizing the proper constraint? iii) How to develop a finite-horizon approach to address the recursive SE problem for MNNs with time delays via DETM under the framework of recursive performance requirement? The corresponding solutions are elaborated as follows: 1) By resorting to the sector-bounded condition, the conditions with respect to the nonlinear activation function are derived in Lemmas 1-2; 2) In the design of the state estimation algorithm, we simultaneously consider two performance constraints, namely the H_∞ performance and the EVB; 3) Sufficient criteria are established to verify that the proposed H_∞ SE method via DETM meets the desired H_∞ performance requirement and the EVB. Specifically, both the disturbance attenuation capability and flexible estimation accuracy are guaranteed through the recursive linear matrix inequalities (RLMIs) technique. The primary innovations are summarized below: i) the dynamic event-triggered H_∞ SE issue is investigated for MNNs subject to time-delay under variance constraint; ii) the DETM is incorporated into the design of time-varying state estimator (TVSE) of MNNs for the purpose of saving energy; and iii) the proposed H_∞ SE algorithm under variance constraints exhibits time-varying characteristics via solving recursive linear matrix inequalities (RLMIs), which is suitable for online applications.

Notations: The superscript T, $\mathbb{E}\{.\}$, $\mathbb{R}^r$,$\text{diag}\{\dots\}$ and $*$ stand for the transpose of the matrix, the mathematical expectation, the r -dimensional Euclidean space, the block diagonal matrix and the ellipsis for term resulting from symmetry, respectively. The full names and abbreviations are given as follows:

Table 1: Definitions of full names and abbreviations.

Full Name	Abbreviation
Neural networks	NNs
Memristive neural networks	MNNs
Dynamic event-triggered mechanism	DETM
State estimation	SE
Estimation error	EE
Time-varying state estimator	TVSE
Error variance boundedness	EVB
Recursive linear matrix inequalities	RLMIs
Positive-definite real-value matrix	PDRVM

2. Problem Formulation

In order to reduce unnecessary waste of computing resources, the DETM is adopted. As illustrated in Fig. (**1**), a dynamic event-triggered H_∞ SE method is proposed for MNNs with variance constraint.

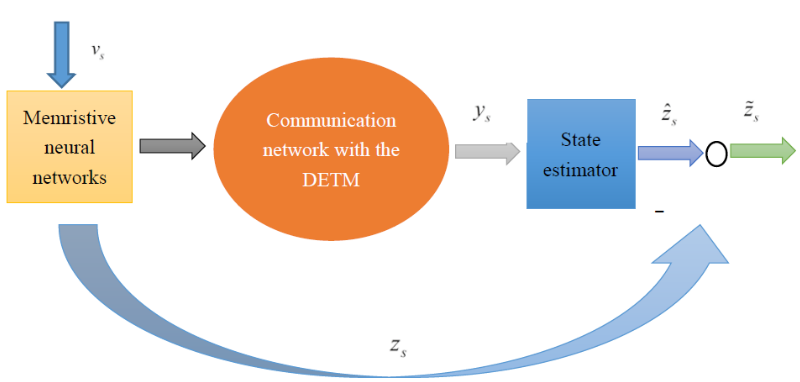

Figure 1: Dynamic event-triggered H_∞ SE for MNNs framework.

In this paper, we consider the MNNs with time-delay described as follows:

$$x_{s+1} = A(x_s)x_s + A_\tau(x_s)x_{s-\tau} + B(x_s)f(x_s) + C_s v_{1s}$$
$$x_s = \phi_s, \forall s \in \{-\tau, -\tau+1, \dots, 0\} \tag{1}$$

where $x_s \in \mathbb{R}^n$ depicts the state vector of MNNs, $A(x_s) = \mathrm{diag}_n\{a_i(x_{i,s})\}$ stands for the state coefficient matrix, $A_\tau(x_s)$ denotes the delayed connection weight matrix, C_s represents known real matrix with suitable dimensions, and $B(x_s)$ depicts the connection weight matrix. ϕ_s denotes a given initial sequence, $f(x_s)$ stands for the nonlinear activation function, and d is the time-delay. v_{1s} denotes zero-mean white noise with covariance $\mathcal{V}_{1s} > 0$.

The activation function $f(\cdot): \mathbb{R}^n \mapsto \mathbb{R}^n$ obeys $f(0) = 0$ and satisfies the sector-bounded condition given as follows:

$$[f(\alpha) - f(\beta) - \mathbf{U}_1(\alpha - \beta)]^T[f(\alpha) - f(\beta) - \mathbf{U}_2(\alpha - \beta)] \le 0,\ \forall \alpha, \beta \in \mathbb{R}^n \tag{2}$$

where $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ depict known matrices, and $\boldsymbol{U} = \boldsymbol{U}_2 - \boldsymbol{U}_1$ stands for the symmetric positive definite real value matrix (PDRVM).

According to [39], the state-dependent functions $a_i(x_{i,s})$, $a_{ij,\tau}(x_{i,s})$ and $b_{ij}(x_{i,s})$ satisfy the following conditions:

$$a_i(x_{i,s}) = \frac{1}{C_i}\left[\sum_{j=1}^{n}\left(\frac{1}{R_{aij,\tau}} + \frac{1}{R_{bij}}\right)\mathrm{sign}_{ij} + \frac{1}{R_i}\right] = \begin{cases}\hat{a}_i, |x_{i,s}| > \Gamma_i \\ \check{a}_i, |x_{i,s}| \le \Gamma_i\end{cases}$$

$$a_{ij,\tau}(x_{i,s}) = \frac{\mathrm{sign}_{ij}}{C_i R_{aij,d}} = \begin{cases}\hat{a}_{ij,\tau}, |x_{i,s}| > \Gamma_i \\ \check{a}_{ij,\tau}, |x_{i,s}| \le \Gamma_i\end{cases}$$

$$b_{ij}(x_{i,s}) = \frac{\mathrm{sign}_{ij}}{C_i R_{bij}} = \begin{cases}\hat{b}_{ij}, |x_{i,s}| > \Gamma_i \\ \check{b}_{ij}, |x_{i,s}| \le \Gamma_i\end{cases}$$

where $\Gamma_i > 0$, $|\hat{a}_i| < 1$, $|\check{a}_i| < 1$, C_i denotes the capacitor, R_i stands for the parallel-resistor, $R_{aij,\tau}$ and R_{bij} are, respectively, the delayed connection weight matrix and the connection weight matrix. $\hat{a}_{ij,\tau}$, $\check{a}_{ij,\tau}$, $\hat{b}_{ij}$ and $\check{b}_{ij}$ are known scalars. Additionally, the symbolic function satisfies the following condition

$$\mathrm{sign}_{ij} = \begin{cases}1, & i \ne j \\ -1, & i = j\end{cases}$$

Denoting

$$a_i^- = min\{\hat{a}_i, \check{a}_i\}, a_i^+ = max\{\hat{a}_i, \check{a}_i\}, a_{ij,\tau}^- = min\{\hat{a}_{ij,\tau}, \check{a}_{ij,\tau}\}$$
$$a_{ij,\tau}^+ = max\{\hat{a}_{ij,\tau}, \check{a}_{ij,\tau}\}, b_{ij}^- = min\{\hat{b}_{ij}, \check{b}_{ij}\}, b_{ij}^+ = max\{\hat{b}_{ij}, \check{b}_{ij}\}$$
$$A^+ = \mathrm{diag}_n\{a_i^+\}, A^- = \mathrm{diag}_n\{a_i^-\}, A_\tau^+ = \{a_{ij,\tau}^+\}_{n\times n}$$
$$A_\tau^- = \{a_{ij,\tau}^-\}_{n\times n}, B^+ = \{b_{ij}^+\}_{n\times n}, B^- = \{b_{ij}^-\}_{n\times n}$$

then we have $A_\tau(x_s) \in [A_\tau^-, A_\tau^+]$, $A(x_s) \in [A^-, A^+]$ and $B(x_s) \in [B^-, B^+]$. By defining $\bar{A}_\tau \triangleq \frac{A_\tau^+ + A_\tau^-}{2} = \left(\frac{a_{ij,\tau}^+ + a_{ij,\tau}^-}{2}\right)_{n\times n}$, $\bar{A} \triangleq \frac{A^+ + A^-}{2} = \mathrm{diag}\left\{\frac{a_1^+ + a_1^-}{2}, \frac{a_2^+ + a_2^-}{2}, \dots, \frac{a_n^+ + a_n^-}{2}\right\}$ and $\bar{B} \triangleq \frac{B^+ + B^-}{2} = \left(\frac{b_{ij}^+ + b_{ij}^-}{2}\right)_{n\times n}$, the matrices $A(x_s)$, $A_\tau(x_s)$ and $B(x_s)$ are further expressed by

$$A(x_s) = \bar{A} + \Delta A_s, A_\tau(x_s) = \bar{A}_\tau + \Delta A_{d\tau}, B(x_s) = \bar{B} + \Delta B_s \tag{3}$$

where $\Delta A_s \in \left[-\frac{A^+ - A^-}{2}, \frac{A^+ - A^-}{2}\right]$, $\Delta A_{d\tau} \in \left[-\frac{A_\tau^+ - A_\tau^-}{2}, \frac{A_\tau^+ - A_\tau^-}{2}\right]$ and $\Delta B_s \in \left[-\frac{B^+ - B^-}{2}, \frac{B^+ - B^-}{2}\right]$. Let $\Delta A_s = \sum_{i=1}^n e_i\, u_{i,s} e_i^T$, $\Delta A_{d\tau} = \sum_{i,j=1}^n e_i\, \psi_{ij,s} e_j^T$ and $\Delta B_s = \sum_{i,j=1}^n e_i\, v_{ij,s} e_j^T$. Here, $e_i \in \mathbb{R}^n$ stands for the column vector with the i -th element being 1 and others being 0. Unknown scalars $u_{i,s}$, $\psi_{ij,s}$ and $v_{ij,s}$ satisfy $|u_{i,s}| \le \tilde{a}_i$, $|\psi_{ij,s}| \le \tilde{a}_{ij,\tau}$ and $|v_{ij,s}| \le \tilde{b}_{ij}$ with $\tilde{a}_i = \frac{a_i^+ - a_i^-}{2}$, $\tilde{a}_{ij,\tau} = \frac{a_{ij,\tau}^+ - a_{ij,\tau}^-}{2}$ and $\tilde{b}_{ij} = \frac{b_{ij}^+ - b_{ij}^-}{2}$. Furthermore, the unknown parameter matrices ΔA_s, $\Delta A_{d\tau}$ and ΔB_s can be written as

$$\Delta A_s = HF_{1,s}N_1, \Delta A_{d\tau} = HF_{2,s}N_2, \Delta B_s = HF_{3,s}N_3$$

where

$$\begin{aligned}
&H = [H_1 \quad H_2 \quad \cdots \quad H_n], H_i = \underbrace{[e_i \quad e_i \quad \cdots \quad e_i]}_{n}, \quad (i = 1,2,\dots,n) \\
&N_l = [N_{l1} \quad N_{l2} \quad \cdots \quad N_{ln}]^T, (l = 1,2,3) \\
&N_{1i} = [e_1 \quad \cdots \quad e_{i-1} \quad \tilde{a}_i e_i \quad e_{i+1} \quad \cdots \quad e_n] \\
&N_{2i} = [\tilde{a}_{i1,\tau}e_1 \quad \tilde{a}_{i2,\tau}e_2 \quad \cdots \quad \tilde{a}_{in,\tau}e_n], N_{3i} = [\tilde{b}_{i1}e_1 \quad \tilde{b}_{i2}e_2 \quad \cdots \quad \tilde{b}_{in}e_n] \\
&F_{l,s} = \text{diag}\{F_{l1,s}, F_{l2,s}, \dots, F_{ln,s}\}, (l = 1,2,3), F_{1i,s} = \text{diag}\{\underbrace{0,\dots,0}_{i-1}, u_{i,s}\tilde{a}_i^{-1}, \underbrace{0,\dots,0}_{n-i}\} \\
&F_{2i,s} = \text{diag}\{\psi_{i1,s}\tilde{a}_{i1,\tau}^{-1}, \psi_{i2,s}\tilde{a}_{i2,\tau}^{-1}, \dots, \psi_{in,s}\tilde{a}_{in,\tau}^{-1}\}, F_{3i,s} = \text{diag}\{v_{i1,s}\tilde{b}_{i1}^{-1}, v_{i2,s}\tilde{b}_{i2}^{-1}, \dots, v_{in,s}\tilde{b}_{in}^{-1}\}
\end{aligned}$$

with H and N_l being known matrices. It is easy to verify that $F_{l,s}$ satisfies $F_{l,s}^T F_{l,s} \le I$.

The measurement output and controlled output are given as follows:

$$\begin{aligned} y_s &= D_s x_s + E_s v_{2s} \\ z_s &= M_s x_s \end{aligned} \tag{4}$$

where $y_s \in \mathbb{R}^m$ stands for the measurement output, $z_s \in \mathbb{R}^r$ stands for the controlled output, D_s, E_s and M_s are known matrices with proper dimensions, and v_{2s} stands for zero-mean white noise with covariance $\mathcal{V}_{2s} > 0$. In what follows, assume that x_0, v_{1s} and v_{2s} are mutually independent.

To save resources, the dynamic event generator is designed between the sensor and the state estimator. Moreover, the triggering instant sequence is denoted by $0 \le t_0 < t_1 < \cdots \le t_l < \cdots$, where t_{l+1} is defined as the following rule

$$t_{l+1} = \min\left\{s \in [0,N] | s > t_l, \frac{1}{\theta}\eta_s + \sigma - \varepsilon_s^T \varepsilon_s \le 0\right\} \tag{5}$$

where $\sigma > 0$ and $\theta > 0$ are given scalars, $\varepsilon_s = y_s - y_{s_{t_l}}$, $y_{s_{t_l}}$ depicts the transmitted measurement at latest event time, and η_s depicts the internal dynamic variable obeying

$$\eta_{s+1} = \lambda\eta_s + \sigma - \varepsilon_s^T \varepsilon_s \tag{6}$$

where $\lambda > 0$ depicts the known constant and $\eta_0 \ge 0$ stands for the known initial condition.

In this paper, the TVSE is designed as follows:

$$\begin{aligned} \hat{x}_{s+1} &= \bar{A}\hat{x}_s + \bar{A}_\tau \hat{x}_{s-\tau} + \bar{B}f(\hat{x}_s) + K_s\left(y_{s_{t_l}} - D_s\hat{x}_s\right) \\ \hat{z}_s &= M_s\hat{x}_s \end{aligned} \tag{7}$$

where K_s is the TVSE gain and $\hat{x}_s$ is the estimation of neural state x_s.

Let the estimation error (EE) be $e_s = x_s - \hat{x}_s$ and the controlled output EE be $\tilde{z}_s = z_s - \hat{z}_s$. Furthermore, the controlled output EE system can be obtained from (1), (4) and (7) as follows:

$$\begin{aligned} e_{s+1} &= (\bar{A} - K_s D_s)e_s + \Delta A_s(e_s + \hat{x}_s) + \bar{A}_\tau e_{s-\tau} + \Delta A_{d\tau}(e_{s-\tau} + \hat{x}_{s-\tau}) + \bar{B}\bar{f}(e_s) + \Delta B_s f(e_s + \hat{x}_s) + C_s v_{1s} - K_s E_s v_{2s} + K_s \varepsilon_s \\ \tilde{z}_s &= M_s e_s \end{aligned} \tag{8}$$

where $\bar{f}(e_s) = f(x_s) - f(\hat{x}_s)$ and $e_{s-\tau} = x_{s-\tau} - \hat{x}_{s-\tau}$.

Subsequently, the EE covariance matrix X_s is specified as follows:

$$X_s = \mathbb{E}\{e_s e_s^T\} \tag{9}$$

The main objective is to construct the TVSE of form (7) for MNNs with variance constraint, and the H_∞ SE algorithm obeys the following requirements simultaneously.

(R1) Let the matrices $\mathcal{U}_\varphi > 0$, $\mathcal{U}_\phi > 0$, $\mathcal{U}_\psi > 0$ and the scalar $\gamma > 0$ be given. The controlled output EE $\tilde{z}_s$ with the initial state $e_l(l = -\tau, -\tau + 1, \dots, 0)$ satisfies

$$J_1 \coloneqq \mathbb{E}\left\{\sum_{s=0}^{N-1}\left(\|\tilde{z}_s\|^2 - \gamma^2\|v_s\|^2_{\mathcal{U}_\varphi}\right)\right\} - \gamma^2\mathbb{E}\{e_0^T\mathcal{U}_\phi e_0 + \sum_{l=-\tau}^{-1} e_l^T\,\mathcal{U}_\psi e_l\} < 0 \tag{10}$$

where $\|v_s\|^2_{\mathcal{U}_\varphi} = v_s^T\mathcal{U}_\varphi v_s$ and $v_s = [v_{1s}^T \quad v_{2s}^T]^T$.

(R2) The EE covariance obeys the condition

$$J_2 \coloneqq X_s \leq \mathfrak{Y}_s \tag{11}$$

where $\mathfrak{Y}_s > 0(0 \leq s \leq N)$ depicts pre-determined known matrix, which reflects the admissible estimation precision demand corresponding to the actual situation.

Remark 1: On the one hand, the non-augmented method designs the state estimator directly based on the original system model, eliminating the need to construct additional augmented states. It avoids the increase in model complexity caused by augmentation. On the other hand, it should be noted that augmented methods need to process both original states and augmented states simultaneously with computational load increasing linearly with the augmented dimension. In contrast, the non-augmented method directly corresponds to the original design objectives and is capable of reducing computational complexity.

Before ending this section, we introduce four lemmas for subsequent calculations.

Lemma 1 [40]: The nonlinear activation function $f(\cdot)$ obeys condition (2), we can deduce

$$\begin{bmatrix} e_s \\ f(e_s + \hat{x}_s) \\ 1 \end{bmatrix}^T \begin{bmatrix} R_{1s} & R_{2s} & R_{3s}^T \\ * & I & -f(\hat{x}_s) \\ * & * & f^T(\hat{x}_s)f(\hat{x}_s) \end{bmatrix} \begin{bmatrix} e_s \\ f(e_s + \hat{x}_s) \\ 1 \end{bmatrix} \leq 0 \tag{12}$$

where

$$R_{1s} -= \frac{\mathbf{U}_1^T\mathbf{U}_2 + \mathbf{U}_2^T\mathbf{U}_1}{2}, R_{2s} = -\frac{\mathbf{U}_1^T + \mathbf{U}_2}{2}, R_{3s} = \frac{f^T(\hat{x}_s)\mathbf{U}_1 + f^T(\hat{x}_s)\mathbf{U}_2}{2}$$

Proof: Based on [40], the proof of this lemma can be easily obtained and is thus omitted here.

Lemma 2: The nonlinear activation function $f(\cdot)$ satisfies condition (2), we obtain

$$f^T(\chi)f(\chi) \leq \left\{\frac{\rho + \frac{1}{\rho}}{2(1-\rho)}\text{tr}(\mathbf{U}_1^T\mathbf{U}_1) + \frac{1}{\rho(1-\rho)}\text{tr}(\mathbf{U}_2^T\mathbf{U}_2)\right\}\|\chi\|^2, \rho \in (0,1) \tag{13}$$

where $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ denote matrices with known appropriate dimensions.

Proof: The derivation of this lemma is straightforward and thus omitted.

Lemma 3 [41]: For the DETM given by (5) and (6) with the initial value $\eta_0 \geq 0$, the internal dynamic variable obeys $\eta_s \geq 0$ for all $s \geq 0$ if the parameters λ $(0 < \lambda < 1)$ and θ $(\theta > 0)$ obey $\lambda\theta \geq 1$.

Proof: Based on the condition (5), it holds that $\frac{1}{\theta}\eta_s + \sigma - \varepsilon_s^T\varepsilon_s \geq 0$ for all $s \geq 0$. Then, it follows from (6) that $\eta_{s+1} \geq \left(\lambda - \frac{\eta_s}{\theta}\right) \geq \cdots \geq \left(\lambda - \frac{1}{\theta}\right)^{s+1}$, which is easily seen $\eta_s > 0$ for all $s \geq 0$ under the condition $\lambda\theta \geq 1$ and $\eta_0 \geq 0$. This completes the proof of Lemma 3.

Lemma 4 [28]: Assume that $\lambda\theta \geq 1$ holds and let scalars $a_s > 0$ and $b_s > 0$ be given. If there exists matrix Y_s obeying

$$\begin{aligned} Y_s &\triangleq \Omega_s(\bar{Y}_s) \\ &\triangleq (1+a_s)(1+b_s)\lambda^2\bar{Y}_s + \left[\frac{(1+a_s^{-1})(1+\theta)^2}{\theta^4} + \frac{(1+\theta)(1+\theta^2)}{\theta^3}\right]\bar{Y}_s^2 \\ &+ \left[(1+a_s)(1+b_s^{-1}) + (1+a_s^{-1})\left(1+\frac{1}{\theta}\right)^2\right]\sigma^2 + \frac{(1+\theta)(1+\theta^2)}{\theta^3}\sigma^4 \end{aligned} \tag{14}$$

with the initial condition $\bar{Y}_s = \eta_0^2$, then $\bar{Y}_s$ is an upper bound of $Y_s \triangleq \mathbb{E}\{\eta_s^2\}$, i.e., $Y_s \leq \bar{Y}_s$.

Proof: Using the inequality $MN^T + MN^T \leq \alpha MM^T + \alpha^{-1}NN^T(\alpha > 0)$, it follows from (5) that

$$\varepsilon_s^T\varepsilon_s \leq \left(\frac{1}{\theta}\eta_s + \sigma\right)^2 \leq \frac{1+\theta}{\theta^2}\eta_s^2 + \left(1+\frac{1}{\theta}\right)\sigma^2 \tag{15}$$

From (6) and (15), we can derive that

$$\begin{aligned} Y_s &= \mathbb{E}\{(\lambda\eta_s + \sigma - \varepsilon_s^T\varepsilon_s)^2\} \\ &= \mathbb{E}\{(\lambda\eta_s + \sigma)^2 + (\varepsilon_s^T\varepsilon_s)^2 - 2(\lambda\eta_s + \sigma)\varepsilon_s^T\varepsilon_s\} \\ &= \mathbb{E}\{\lambda^2\eta_s^2 + \sigma^2 + 2\lambda\eta_s\sigma + (\varepsilon_s^T\varepsilon_s)^2 - 2(\lambda\eta_s + \sigma)\varepsilon_s^T\varepsilon_s\} \\ &\leq (1+a_s)(1+b_s)\lambda^2\mathbb{E}\{\eta_s^2\} + (1+a_s)(1+b_s^{-1})\sigma^2 + (1+a_s^{-1})\mathbb{E}\{(\varepsilon_s^T\varepsilon_s)^2\} \\ &\leq (1+a_s)(1+b_s)\lambda^2\mathbb{E}\{\eta_s^2\} + (1+a_s)(1+b_s^{-1})\sigma^2 \\ &+(1+a_s^{-1})\left[\frac{(1+\theta)^2}{\theta^4}\eta_s^4 + \left(1+\frac{1}{\theta}\right)^2\sigma^4 + \frac{2(1+\theta)(\theta^2+1)}{\theta^3}\eta_s^2\sigma^2\right] \\ &\leq (1+a_s)(1+b_s)\lambda^2 Y_s + \left[\frac{(1+a_s^{-1})(1+\theta)^2}{\theta^4} + \frac{(1+\theta)(1+\theta^2)}{\theta^3}\right]Y_s^2 \\ &+\left[\frac{(1+\theta)(1+\theta^2)}{\theta^3} + (1+a_s)(1+b_s^{-1})\right]\sigma^2 + (1+a_s^{-1})\left(1+\frac{1}{\theta}\right)^2\sigma^4 \end{aligned}$$

Furthermore, we can easily obtain $Y_s \leq \bar{Y}_s$, which ends the proof.

3. Primary Results

In this section, new sufficient conditions are derived to guarantee two desirable performance indices including the prescribed *H*∞ performance requirement and the EVB.

3.1. *H*∞ Performance Analysis

To begin with, a sufficient condition is obtained to ensure the *H*∞ performance constraint via the RLMIs method.

Theorem 1: Consider the MNNs with variance constraint (1). Suppose that matrices $\mathcal{U}_\varphi > 0$, $\mathcal{U}_\phi > 0$ and $\mathcal{U}_\psi > 0$, the scalar $\gamma > 0$ and the TVSE gain matrix K_s in (7) are given. Under initial conditions $\eta_0 = 0$, $\mathcal{R}_0 \leq \gamma^2\mathcal{U}_\phi$ and $\mathfrak{Q}_l \leq \gamma^2\mathcal{U}_\psi(l = -\tau, -\tau+1, \ldots, -1)$, if there exist PDRVMs $\{\mathcal{R}_s\}_{1\leq s\leq N+1}$, $\{\mathfrak{Q}_s\}_{0\leq s\leq N}$ and the positive scalar κ_s obeying the inequality

$$\Theta=\begin{bmatrix}\Theta_{11} & \Theta_{12} & -R_{3s}^T & 0 & 0 & 0 & 0 & 0\\ * & \Theta_{22} & f(\hat{x}_s) & 0 & 0 & 0 & 0 & 0\\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Theta_{44} & 0 & 0 & 0 & 0\\ * & * & * & * & \Theta_{55} & 0 & 0 & 0\\ * & * & * & * & * & \Theta_{66} & 0 & 0\\ * & * & * & * & * & * & \Theta_{77} & 0\\ * & * & * & * & * & * & * & \Theta_{88}\end{bmatrix}<0 \tag{16}$$

with

$$\begin{aligned}
\Theta_{11} &= 10\bar{A}^T\mathcal{R}_{s+1}\bar{A}+11D_s^TK_s^T\mathcal{R}_{s+1}K_sD_s+11\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s+\mathfrak{Q}_s+M_s^TM_s-\mathcal{R}_s-R_{1s}\\
\Theta_{12} &= \bar{A}^T\mathcal{R}_{s+1}\bar{B}-R_{2s},\Theta_{22}=11\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s+10\bar{B}^T\mathcal{R}_{s+1}\bar{B}-I\\
\Theta_{33} &= 11\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s+11\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau}+11f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+\frac{\sigma}{\theta}-\kappa_s\sigma-f^T(\hat{x}_s)f(\hat{x}_s)\\
\Theta_{44} &= 11\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{A}_\tau+11\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}-\mathfrak{Q}_{s-\tau}\\
\Theta_{55} &= 12K_s^T\mathcal{R}_{s+1}K_s-\left(\frac{1}{\theta}+\kappa_s\right)I,\Theta_{66}=\frac{\lambda-1+\kappa_s}{\theta}I\\
\Theta_{77} &= C_s^T\mathcal{R}_{s+1}C_s-\gamma^2\mathcal{U}_\varphi,\Theta_{88}=2E_s^TK_s^T\mathcal{R}_{s+1}K_sE_s-\gamma^2\mathcal{U}_\varphi
\end{aligned} \tag{17}$$

then the H_∞ performance constraint in (10) is ensured.

Proof: Define

$$\mathcal{M}(e_s)=e_s^T\mathcal{R}_se_s+\sum_{l=s-\tau}^{s-1}e_l^T\mathfrak{Q}_le_l+\frac{\eta_s}{\theta} \tag{18}$$

To proceed, according to the EE system (8), it can be concluded that

$$\begin{aligned}
\mathbb{E}\{\Delta\mathcal{M}(e_s)\} = \mathbb{E}\{ & e_s^T\bar{A}^T\mathcal{R}_{s+1}\bar{A}e_s+e_s^TD_s^TK_s^T\mathcal{R}_{s+1}K_sD_se_s+e_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_se_s+\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s+e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau}\\
&+f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)+e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}+\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau}+f^T(e_s+\hat{x}_s)\Delta B_s^T\\
&\times\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+v_{1s}^TC_s^T\mathcal{R}_{s+1}C_sv_{1s}+v_{2s}^TE_s^TK_s^T\mathcal{R}_{s+1}K_sE_sv_{2s}+f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+\varepsilon_s^TK_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\\
&-2e_s^T\bar{A}^T\mathcal{R}_{s+1}K_sD_se_s+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\Delta A_se_s+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau}+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)\\
&+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau}+2e_s^T\bar{A}^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+2e_s^T\bar{A}^T\mathcal{R}_{s+1}K_s\varepsilon_s-2e_s^T\bar{A}^T\mathcal{R}_{s+1}\\
&\times\bar{B}f(\hat{x}_s)-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\Delta A_se_s-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau}-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)\\
&-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)-2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\\
&+2e_s^TD_s^TK_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau}+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\\
&\times\Delta A_{d\tau}e_{s-\tau}+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+2e_s^T\Delta A_s^T\mathcal{R}_{s+1}K_s\varepsilon_s-2e_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\\
&+2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau}+2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)+2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}+2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}+2\hat{x}_s^T\Delta A_s^T\\
&\times\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}K_s\varepsilon_s-2\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s)+2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\\
&\times\Delta A_{d\tau}e_{s-\tau}+2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}+2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}K_s\varepsilon_s-2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\\
&+2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}+2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}+2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)\\
&+2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}K_s\varepsilon_s-2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}+2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta B_s\\
&\times f(e_s+\hat{x}_s)+2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}K_s\varepsilon_s-2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta B_sf(e_s+\hat{x}_s)+2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\\
&\times K_s\varepsilon_s-2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)+2f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}K_s\varepsilon_s-2f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)-2\varepsilon_s^TK_s^T\mathcal{R}_{s+1}\\
&\times\bar{B}f(\hat{x}_s)-2v_{2s}^TE_s^TK_s^T\mathcal{R}_{s+1}K_s\varepsilon_s+\frac{1}{\theta}(\lambda\eta_s+\sigma-\varepsilon_s^T\varepsilon_s)-\frac{\eta_s}{\theta}+e_s^T\mathfrak{Q}_se_s-e_{s-\tau}^T\mathfrak{Q}_{s-\tau}e_{s-\tau}-e_s^T\mathcal{R}_se_s\Big\}
\end{aligned} \tag{19}$$

Applying the fundamental inequality $2a^T Hb \leq a^T Ha + b^T Hb (H > 0)$, we can derive the following results

$$\mathbb{E}\{-2e_s^T \bar{A}^T \mathcal{R}_{s+1} K_s D_s e_s\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + e_s^T D_s^T K_s^T \mathfrak{Q}_{k+1} K_s D_s e_s\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \Delta A_s e_s\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + \hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + e_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \Delta A_{d\tau} \hat{x}_{s-\tau}\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + \hat{x}_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} \hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + f^T(e_s + \hat{x}_s) \Delta B_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2e_s^T \bar{A}^T \mathcal{R}_{s+1} K_s \varepsilon_s\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + \varepsilon_s^T K_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\}$$

$$\mathbb{E}\{-2e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\} \leq \mathbb{E}\{e_s^T \bar{A}^T \mathcal{R}_{s+1} \bar{A} e_s + f^T(\hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B}_s f(\hat{x}_s)\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \Delta A_s e_s\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + \hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + f^T(e_s + \hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + e_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + \hat{x}_{s-\tau}^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + f^T(e_s + \hat{x}_s) \Delta B_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{-2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + \varepsilon_s^T K_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\}$$

$$\mathbb{E}\{2e_s^T D_s^T K_s^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\} \leq \mathbb{E}\{e_s^T D_s^T K_s^T \mathcal{R}_{s+1} K_s D_s e_s + f^T(\hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + \hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + f^T(e_s + \hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + e_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + \hat{x}_{s-\tau}^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + f^T(e_s + \hat{x}_s) \Delta B_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2e_s^T \Delta A_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + \varepsilon_s^T K_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\}$$

$$\mathbb{E}\{-2e_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\} \leq \mathbb{E}\{e_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s e_s + f^T(\hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau}\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + f^T(e_s + \hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + e_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + \hat{x}_{s-\tau}^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + f^T(e_s + \hat{x}_s) \Delta B_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + \varepsilon_s^T K_s^T \mathcal{R}_{s+1} K_s \varepsilon_s\}$$

$$\mathbb{E}\{-2\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\} \leq \mathbb{E}\{\hat{x}_s^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_s + f^T(\hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(\hat{x}_s)\}$$

$$\mathbb{E}\{2e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau} + f^T(e_s + \hat{x}_s) \bar{B}^T \mathcal{R}_{s+1} \bar{B} f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\} \leq \mathbb{E}\{e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau} + e_{s-\tau}^T \Delta A_{d\tau}^T \mathcal{R}_{s+1} \Delta A_{d\tau} e_{s-\tau}\}$$

$$\mathbb{E}\{2e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\} \leq \mathbb{E}\{e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau} + \hat{x}_{s-\tau}^T \Delta A_s^T \mathcal{R}_{s+1} \Delta A_s \hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\} \leq \mathbb{E}\{e_{s-\tau}^T \bar{A}_\tau^T \mathcal{R}_{s+1} \bar{A}_\tau e_{s-\tau} + f^T(e_s + \hat{x}_s) \Delta B_s^T \mathcal{R}_{s+1} \Delta B_s f(e_s + \hat{x}_s)\}$$

$$\mathbb{E}\{2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau} + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

$$\mathbb{E}\{-2e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau} + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau}\}$$

$$\mathbb{E}\{2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + \hat{x}_{s-\tau}^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\}$$

$$\mathbb{E}\{2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

$$\mathbb{E}\{-2f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}\} \le \mathbb{E}\{e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau} + \hat{x}_{s-\tau}^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_{s-\tau}\}$$

$$\mathbb{E}\{2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\} \le \mathbb{E}\{e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau} + f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\}$$

$$\mathbb{E}\{2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau} + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

$$\mathbb{E}\{-2e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau} + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\} \le \mathbb{E}\{\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau} + f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s)\}$$

$$\mathbb{E}\{-2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau} + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{2\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau} + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

$$\mathbb{E}\{2f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s) + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

$$\mathbb{E}\{-2f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(\hat{x}_s) + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{-2\varepsilon_s^T K_s^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\} \le \mathbb{E}\{\varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s + f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s)\}$$

$$\mathbb{E}\{-2v_{2s}^T E_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\} \le \mathbb{E}\{v_{2s}^T E_s^T K_s^T\mathcal{R}_{s+1}K_s E_s v_{2s} + \varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s\}$$

Furthermore, through systematic collation and integration of the aforementioned analysis, the following results are derived:

$$\begin{aligned}\mathbb{E}\{\Delta\mathcal{M}(e_s)\} \le \mathbb{E}\{&10e_s^T\bar{A}^T\mathcal{R}_{s+1}\bar{A}e_s + 11e_s^T D_s^T K_s^T\mathcal{R}_{s+1}K_s D_s e_s + 11e_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s e_s + 11\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s \\ &+11e_{s-\tau}^T\bar{A}_\tau^T\mathcal{R}_{s+1}\bar{A}_\tau e_{s-\tau} + 10f^T(e_s+\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) + 11e_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}e_{s-\tau} \\ &+11\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau} + 11f^T(e_s+\hat{x}_s)\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s f(e_s+\hat{x}_s) + v_{1s}^T C_s^T\mathcal{R}_{s+1}C_s v_{1s} \\ &+2v_{2s}^T E_s^T K_s^T\mathcal{R}_{s+1}K_s E_s v_{2s} + 11f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s) + 12\varepsilon_s^T K_s^T\mathcal{R}_{s+1}K_s\varepsilon_s + 2e_s^T\bar{A}^T\mathcal{R}_{s+1}\bar{B}f(e_s+\hat{x}_s) \\ &+e_s^T\mathfrak{Q}_s e_s - e_{s-\tau}^T\mathfrak{Q}_{s-\tau}e_{s-\tau} - e_s^T\mathcal{R}_s e_s + \frac{1}{\theta}(\lambda\eta_s + \sigma - \varepsilon_s^T\varepsilon_s) - \frac{\eta_s}{\theta}\Big\}\end{aligned} \quad (20)$$

Adding the zero term $\tilde{z}_s^T\tilde{z}_s - \gamma^2 v_s^T\mathcal{U}_\varphi v_s - \tilde{z}_s^T\tilde{z}_s + \gamma^2 v_s^T\mathcal{U}_\varphi v_s$ to $\mathbb{E}\{\Delta\mathcal{M}(e_s)\}$, it is straightforward to obtain

$$\mathbb{E}\{\Delta\mathcal{M}(e_s)\} \le \mathbb{E}\left\{[\Pi_s^T \quad v_s^T]\widetilde{\Theta}\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix} - \tilde{z}_s^T\tilde{z}_s + \gamma^2 v_s^T\mathcal{U}_\varphi v_s\right\} \quad (21)$$

where

$$\Pi_s = \left[e_s^T \quad f^T(e_s+\hat{x}_s) \quad 1 \quad e_{s-\tau}^T \quad \varepsilon_s^T \quad (\eta_s^{\frac{1}{2}})^T\right]^T$$

$$\widetilde{\Theta} = \begin{bmatrix} \widetilde{\Theta}_{11} & \bar{A}^T\mathcal{R}_{s+1}\bar{B} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \widetilde{\Theta}_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \widetilde{\Theta}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Theta_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Theta}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \widetilde{\Theta}_{66} & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 \\ * & * & * & * & * & * & * & \Theta_{88} \end{bmatrix}$$

$$\begin{aligned}
\widetilde{\Theta}_{11} &= 10\bar{A}^T\mathcal{R}_{s+1}\bar{A} + 11D_s^TK_s^T\mathcal{R}_{s+1}K_sD_s + 11\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s + \mathfrak{Q}_s + M_s^TM_s - \mathcal{R}_s \\
\widetilde{\Theta}_{22} &= 11\Delta B_s^T\mathcal{R}_{s+1}\Delta B_s + 10\bar{B}^T\mathcal{R}_{s+1}\bar{B} \\
\widetilde{\Theta}_{33} &= 11\hat{x}_s^T\Delta A_s^T\mathcal{R}_{s+1}\Delta A_s\hat{x}_s + 11\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\mathcal{R}_{s+1}\Delta A_{d\tau}\hat{x}_{s-\tau} + 11f^T(\hat{x}_s)\bar{B}^T\mathcal{R}_{s+1}\bar{B}f(\hat{x}_s) + \frac{\sigma}{\theta} \\
\widetilde{\Theta}_{55} &= 12K_s^T\mathcal{R}_{s+1}K_s - \frac{1}{\theta}I, \widetilde{\Theta}_{66} = \frac{\lambda-1}{\theta}I
\end{aligned} \tag{22}$$

with Θ_{44}, Θ_{77} and Θ_{88} defined below (17).

Based on Lemma 1 and (5), it follows that

$$\begin{aligned}
\mathbb{E}\{\Delta\mathcal{M}(e_s)\} &\le \mathbb{E}\left\{[\Pi_s^T \quad v_s^T]\widetilde{\Theta}\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix} - \tilde{z}_s^T\tilde{z}_s + \gamma^2v_s^T\mathcal{U}_\varphi v_s - [e_s^TR_{1s}e_s + 2e_s^TR_{2s}f(\hat{x}_s+e_s) + 2e_s^TR_{3s}^T\right. \\
&\quad \left. -2f^T(\hat{x}_s+e_s)f(\hat{x}_s) + f^T(\hat{x}_s+e_s)f(\hat{x}_s+e_s) + f^T(\hat{x}_s)f(\hat{x}_s)] + \kappa_s\left(\frac{\eta_s}{\theta} + \sigma - \varepsilon_s^T\varepsilon_s\right)\right\} \\
&= \mathbb{E}\left\{[\Pi_s^T \quad v_s^T]\Theta\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix} - \tilde{z}_s^T\tilde{z}_s + \gamma^2v_s^T\mathcal{U}_\varphi v_s\right\}
\end{aligned} \tag{23}$$

where Θ is defined in (16) and κ_s depicts a positive scalar.

Summing both sides of (23) with respect to s from 0 to $N-1$, it is straightforward to get

$$\begin{aligned}
\sum_{s=0}^{N-1}\mathbb{E}\{\Delta\mathcal{M}(e_s)\} &= \mathbb{E}\left\{e_N^T\mathcal{R}_Ne_N - e_0^T\mathcal{R}_0e_0 + \sum_{l=N-\tau}^{N-1}e_l^T\mathfrak{Q}_le_l - \sum_{l=-\tau}^{-1}e_l^T\mathfrak{Q}_le_l + \frac{\eta_N}{\theta}\right\} \\
&\le \mathbb{E}\left\{\sum_{s=0}^{N-1}[\Pi_s^T \quad v_s^T]\,\Theta\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix}\right\} - \mathbb{E}\left\{\sum_{s=0}^{N-1}\left(\tilde{z}_s^T\tilde{z}_s - \gamma^2v_s^T\mathcal{U}_\varphi v_s\right)\right\}
\end{aligned} \tag{24}$$

Furthermore, we obtain

$$\begin{aligned}
J_1 &= \mathbb{E}\left\{\sum_{s=0}^{N-1}\left(\|\tilde{z}_s\|^2 - \gamma^2\|v_s\|_{\mathcal{U}_\varphi}^2\right)\right\} - \gamma^2\mathbb{E}\left\{e_0^T\mathcal{U}_\phi e_0 + \sum_{l=-\tau}^{-1}e_l^T\mathcal{U}_\psi e_l\right\} \\
&\le -\mathbb{E}\left\{e_N^T\mathcal{R}_Ne_N - e_0^T\mathcal{R}_0e_0 + \sum_{l=N-\tau}^{N-1}e_l^T\mathfrak{Q}_le_l - \sum_{l=-\tau}^{-1}e_l^T\mathfrak{Q}_le_l + \frac{\eta_N}{\theta}\right\} \\
&\quad -\gamma^2\mathbb{E}\left\{e_0^T\mathcal{U}_\phi e_0 + \sum_{l=-\tau}^{-1}e_l^T\mathcal{U}_\psi e_l\right\} + \mathbb{E}\left\{\sum_{s=0}^{N-1}[\Pi_s^T \quad v_s^T]\,\Theta\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix}\right\} \\
&= \mathbb{E}\left\{\sum_{s=0}^{N-1}[\Pi_s^T \quad v_s^T]\,\Theta\begin{bmatrix}\Pi_s \\ v_s\end{bmatrix} + e_0^T(\mathcal{R}_0 - \gamma^2\mathcal{U}_\phi)e_0 + \sum_{l=-\tau}^{-1}e_l^T(\mathfrak{Q}_l - \gamma^2\mathcal{U}_\psi)e_l\right\} \\
&\quad -\mathbb{E}\left\{e_N^T\mathcal{R}_Ne_N + \sum_{l=N-\tau}^{N-1}e_l^T\mathfrak{Q}_le_l + \frac{\eta_N}{\theta}\right\}
\end{aligned} \tag{25}$$

From the conditions $\Theta < 0$, $\mathcal{R}_N > 0$, $\mathfrak{Q}_l > 0$, $\mathcal{R}_0 \le \gamma^2\mathcal{U}_\phi$, $\eta_N > 0$ and $\mathfrak{Q}_l \le \gamma^2\mathcal{U}_\psi (l = -\tau, -\tau+1, \ldots, -1)$, it is straightforward to derive $J_1 < 0$.

3.2. Variance Constraint Analysis

Subsequently, we begin to analyze the variance constraint, that is, the sufficient condition is derived to guarantee the EVB by using the stochastic analysis technique.

Theorem 2: Consider the MNNs with variance constraint (1). Given the TVSE gain matrix K_s in (7), under the initial condition $\mathcal{G}_0 = X_0$, if there exist PDRVMs $\{\mathcal{G}_s\}_{1\le s\le N+1}$ satisfying the inequality

$$\mathcal{G}_{s+1} \ge \mathfrak{X}(\mathcal{G}_s) \tag{26}$$

where

$$\begin{aligned}\mathfrak{X}(\mathcal{G}_s) &= 10\bar{A}\mathcal{G}_s\bar{A}^T + 10K_sD_s\mathcal{G}_sD_s^TK_s^T + 10\Delta A_s\mathcal{G}_s\Delta A_s^T + 10\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T + 10\bar{A}_\tau\mathcal{G}_{s-\tau}\bar{A}_\tau^T + 10\mathfrak{X}\mathrm{tr}(\mathcal{G}_s)\bar{B}\bar{B}^T \\ &+10\Delta A_{d\tau}\mathcal{G}_{s-\tau}\Delta A_{d\tau}^T + 10\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T + 20\mathfrak{X}\mathrm{tr}(\mathcal{G}_s)\Delta B_s\Delta B_s^T + 20\mathfrak{X}\mathrm{tr}(\hat{x}_s\hat{x}_s^T)\Delta B_s\Delta B_s^T + 11\varpi K_sK_s^T \\ &+C_s\mathcal{V}_{1s}C_s^T + 2K_sE_s\mathcal{V}_{2s}E_s^TK_s^T, \mathfrak{X} = \frac{\rho+\frac{1}{\rho}}{2(1-\rho)}\mathrm{tr}(\mathbf{U}_1^T\mathbf{U}_1) + \frac{1}{\rho(1-\rho)}\mathrm{tr}(\mathbf{U}_2^T\mathbf{U}_2) \\ \varpi &= \frac{1+\theta}{\theta^2}\eta_s^2 + \left(1+\frac{1}{\theta}\right)\sigma^2\end{aligned} \tag{27}$$

then it follows that $\mathcal{G}_s \ge X_s \forall s \in \{1,2,\dots,N+1\}$.

Proof: In light of (9), the EE covariance matrix X_{s+1} can be derived as:

$$\begin{aligned}X_{s+1} &= \mathbb{E}\{e_{s+1}e_{s+1}^T\} \\ &= \mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T + K_sD_se_se_s^TD_s^TK_s^T + \Delta A_se_se_s^T\Delta A_s^T + \Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T + \bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T + \bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T \\ &+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T + \Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T + C_sv_{1s}v_{1s}^TC_s^T + K_sE_sv_{2s}v_{2s}^T \\ &\times E_s^TK_s^T + K_s\varepsilon_s\varepsilon_s^TK_s^T - K_sD_se_se_s^T\bar{A}^T - \bar{A}e_se_s^TD_s^TK_s^T + \Delta A_se_se_s^T\bar{A}^T + \bar{A}e_se_s^T\Delta A_s^T + \Delta A_s\hat{x}_se_s^T\bar{A}^T + \bar{A}e_s \\ &\times \hat{x}_s^T\Delta A_s^T + \bar{A}_\tau e_{s-\tau}e_s^T\bar{A}^T + \bar{A}e_se_{s-\tau}^T\bar{A}_\tau^T + \bar{B}\bar{f}(e_s)e_s^T\bar{A}^T + \bar{A}e_s\bar{f}^T(e_s)\bar{B}^T + \Delta A_{d\tau}e_{s-\tau}e_s^T\bar{A}^T + \bar{A}e_se_{s-\tau}^T\Delta A_{d\tau}^T \\ &+\Delta A_{d\tau}\hat{x}_{s-\tau}e_s^T\bar{A}^T + \bar{A}e_s\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T + \Delta B_sf(e_s+\hat{x}_s)e_s^T\bar{A}^T + \bar{A}e_sf^T(e_s+\hat{x}_s)\Delta B_s^T + K_s\varepsilon_se_s^T\bar{A}^T + \bar{A}e_s\varepsilon_s^TK_s^T \\ &-\Delta A_se_se_s^TD_s^TK_s^T - K_sD_se_se_s^T\Delta A_s^T - \Delta A_s\hat{x}_se_s^TD_s^TK_s^T - K_sD_se_s\hat{x}_s^T\Delta A_s^T - \bar{A}_\tau e_{s-\tau}e_s^TD_s^TK_s^T - K_sD_se_se_{s-\tau}^T\bar{A}_\tau^T \\ &-\bar{B}\bar{f}(e_s)e_s^TD_s^TK_s^T - K_sD_se_s\bar{f}^T(e_s)\bar{B}^T - \Delta A_{d\tau}e_{s-\tau}e_s^TD_s^TK_s^T - K_sD_se_se_{s-\tau}^T\Delta A_{d\tau}^T - \Delta A_s\hat{x}_{s-\tau}e_s^TD_s^TK_s^T - K_sD_s \\ &\times e_s\hat{x}_{s-\tau}^T\Delta A_s^T - \Delta B_sf(e_s+\hat{x}_s)e_s^TD_s^TK_s^T - K_sD_se_sf^T(e_s+\hat{x}_s)\Delta B_s^T - K_s\varepsilon_se_s^TD_s^TK_s^T - K_sD_se_s\varepsilon_s^TK_s^T + \Delta A_s \\ &\times \hat{x}_se_s^T\Delta A_s^T + \Delta A_se_s\hat{x}_s^T\Delta A_s^T + \bar{A}_\tau e_{s-\tau}e_s^T\Delta A_s^T + \Delta A_se_se_{s-\tau}^T\bar{A}_\tau^T + \bar{B}\bar{f}(e_s)e_s^T\Delta A_s^T + \Delta A_se_s\bar{f}^T(e_s)\bar{B}^T + \Delta A_{d\tau}e_{s-\tau}e_s^T \\ &\times \Delta A_s^T + \Delta A_se_se_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_s\hat{x}_{s-\tau}e_s^T\Delta A_s^T + \Delta A_se_s\hat{x}_{s-\tau}^T\Delta A_s^T + \Delta B_sf(e_s+\hat{x}_s)e_s^T\Delta A_s^T + \Delta A_se_sf^T(e_s+\hat{x}_s)\Delta B_s^T \\ &+K_s\varepsilon_se_s^T\Delta A_s^T + \Delta A_se_s\varepsilon_s^TK_s^T + \bar{A}_\tau e_{s-\tau}\hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_se_{s-\tau}^T\bar{A}_\tau^T + \bar{B}\bar{f}(e_s)\hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_s\bar{f}^T(e_s)\bar{B}^T + \Delta A_{d\tau}e_{s-\tau} \\ &\times \hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_se_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_s\hat{x}_{s-\tau}\hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_s\hat{x}_{s-\tau}^T\Delta A_s^T + \Delta B_sf(e_s+\hat{x}_s)\hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_sf^T(e_s+\hat{x}_s) \\ &\times \Delta B_s^T + K_s\varepsilon_s\hat{x}_s^T\Delta A_s^T + \Delta A_s\hat{x}_s\varepsilon_s^TK_s^T + \bar{B}\bar{f}(e_s)e_{s-\tau}^T\bar{A}_\tau^T + \bar{A}_\tau e_{s-\tau}\bar{f}^T(e_s)\bar{B}^T + \Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T + \bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T \\ &+\Delta A_s\hat{x}_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T + \bar{A}_\tau e_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T + \Delta B_sf(e_s+\hat{x}_s)e_{s-\tau}^T\bar{A}_\tau^T + \bar{A}_\tau e_{s-\tau}f^T(e_s+\hat{x}_s)\Delta B_s^T + K_s\varepsilon_se_{s-\tau}^T\bar{A}_\tau^T + \bar{A}_\tau e_{s-\tau} \\ &\times \varepsilon_s^TK_s^T + \Delta A_{d\tau}e_{s-\tau}\bar{f}^T(e_s)\bar{B}^T + \bar{B}\bar{f}(e_s)e_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_s\hat{x}_{s-\tau}\bar{f}^T(e_s)\bar{B}^T + \bar{B}\bar{f}(e_s)\hat{x}_{s-\tau}^T\Delta A_s^T + \Delta B_sf(e_s+\hat{x}_s) \\ &\times \bar{f}^T(e_s)\bar{B}^T + \bar{B}\bar{f}(e_s)f^T(e_s+\hat{x}_s)\Delta B_s^T + K_s\varepsilon_s\bar{f}^T(e_s)\bar{B}^T + \bar{B}\bar{f}(e_s)\varepsilon_s^TK_s^T + \Delta A_s\hat{x}_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}e_{s-\tau}\hat{x}_{s-\tau}^T \\ &\times \Delta A_s^T + \Delta B_sf(e_s+\hat{x}_s)e_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}e_{s-\tau}f^T(e_s+\hat{x}_s)\Delta B_s^T + K_s\varepsilon_se_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}e_{s-\tau}\varepsilon_s^TK_s^T + \Delta B_sf(e_s+\hat{x}_s) \\ &\times \hat{x}_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}\hat{x}_{s-\tau}f^T(e_s+\hat{x}_s)\Delta B_s^T + K_s\varepsilon_s\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T + \Delta A_{d\tau}\hat{x}_{s-\tau}\varepsilon_s^TK_s^T + K_s\varepsilon_sf^T(e_s+\hat{x}_s)\Delta B_s^T + \Delta B_sf(e_s+\hat{x}_s) \\ &\times \varepsilon_s^TK_s^T - K_s\varepsilon_sv_{2s}^TE_s^TK_s^T - K_sE_sv_{2s}\varepsilon_s^TK_s^T\}\end{aligned} \tag{28}$$

Based on the inequality $xy^T + yx^T \le xx^T + yy^T$, the following result can be derived

$$\mathbb{E}\{-K_sD_se_se_s^T\bar{A}^T - \bar{A}e_se_s^TD_s^TK_s^T\} \le \mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T + K_sD_se_se_s^TD_s^TK_s^T\}$$
$$\mathbb{E}\{\Delta A_se_se_s^T\bar{A}^T + \bar{A}e_se_s^T\Delta A_s^T\} \le \mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T + \Delta A_se_se_s^T\Delta A_s^T\}$$

$$\mathbb{E}\{\Delta A_s\hat{x}_s e_s^T\bar{A}^T+\bar{A}e_s\hat{x}_s^T\Delta A_s^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\bar{A}_d\hat{x}_s\hat{x}_s^T\bar{A}_d^T\}$$
$$\mathbb{E}\{\bar{A}_\tau e_{s-\tau}e_s^T\bar{A}^T+\bar{A}e_se_{s-\tau}^T\bar{A}_\tau^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\}$$
$$\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_s^T\bar{A}^T+\bar{A}e_se_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta A_{d\tau}\hat{x}_{s-\tau}e_s^T\bar{A}^T+\bar{A}e_s\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta B_sf(e_s+\hat{x}_s)e_s^T\bar{A}^T+\bar{A}e_sf^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{\bar{B}\bar{f}(e_s)e_s^T\bar{A}^T+\bar{A}e_s\bar{f}^T(e_s)\bar{B}^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T\}$$
$$\mathbb{E}\{K_s\varepsilon_se_s^T\bar{A}^T+\bar{A}e_s\varepsilon_s^TK_s^T\}\le\mathbb{E}\{\bar{A}e_se_s^T\bar{A}^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{-\Delta A_se_se_s^TD_s^TK_s^T-K_sD_se_se_s^T\Delta A_s^T\mathcal{R}_{s+1}\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\Delta A_se_se_s^T\Delta A_s^T\}$$
$$\mathbb{E}\{-\Delta A_s\hat{x}_se_s^TD_s^TK_s^T-K_sD_se_s\hat{x}_s^T\Delta A_s^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T\}$$
$$\mathbb{E}\{-\bar{A}_\tau e_{s-\tau}e_s^TD_s^TK_s^T-K_sD_se_se_{s-\tau}^T\bar{A}_\tau^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\}$$
$$\mathbb{E}\{-\bar{B}\bar{f}(e_s)e_s^TD_s^TK_s^T-K_sD_se_s\bar{f}^T(e_s)\bar{B}^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T\}$$
$$\mathbb{E}\{-\Delta A_{d\tau}e_{s-\tau}e_s^TD_s^TK_s^T-K_sD_se_se_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{-\Delta A_s\hat{x}_{s-\tau}e_s^TD_s^TK_s^T-K_sD_se_s\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$
$$\mathbb{E}\{-\Delta B_sf(e_s+\hat{x}_s)e_s^TD_s^TK_s^T-K_sD_se_sf^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{-K_s\varepsilon_se_s^TD_s^TK_s^T-K_sD_se_s\varepsilon_s^TK_s^T\}\le\mathbb{E}\{K_sD_se_se_s^TD_s^TK_s^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_se_s^T\Delta A_s^T+\Delta A_se_s\hat{x}_s^T\Delta A_s^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T\}$$
$$\mathbb{E}\{\bar{A}_\tau e_{s-\tau}e_s^T\Delta A_s^T+\Delta A_se_se_{s-\tau}^T\bar{A}_\tau^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\}$$
$$\mathbb{E}\{\bar{B}\bar{f}(e_s)e_s^T\Delta A_s^T+\Delta A_se_s\bar{f}^T(e_s)\bar{B}^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T\}$$
$$\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_s^T\Delta A_s^T+\Delta A_se_se_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_{s-\tau}e_s^T\Delta A_s^T+\Delta A_se_s\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$
$$\mathbb{E}\{\Delta B_sf(e_s+\hat{x}_s)e_s^T\Delta A_s^T+\Delta A_se_sf^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_se_s^T\Delta A_s^T+\Delta A_se_s\varepsilon_s^TK_s^T\}\le\mathbb{E}\{\Delta A_se_se_s^T\Delta A_s^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{\bar{A}_\tau e_{s-\tau}\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_se_{s-\tau}^T\bar{A}_\tau^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\}$$
$$\mathbb{E}\{\bar{B}\bar{f}(e_s)\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_s\bar{f}^T(e_s)\bar{B}^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T\}$$
$$\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_se_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_{s-\tau}\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_s\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$
$$\mathbb{E}\{\Delta B_sf(e_s+\hat{x}_s)\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_sf^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_s\hat{x}_s^T\Delta A_s^T+\Delta A_s\hat{x}_s\varepsilon_s^TK_s^T\}\le\mathbb{E}\{\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{\bar{B}\bar{f}(e_s)e_{s-\tau}^T\bar{A}_\tau^T+\bar{A}_\tau e_{s-\tau}\bar{f}^T(e_s)\bar{B}^T\}\le\mathbb{E}\left\{\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\bar{B}\bar{f}(e_s)\overline{f^T}(e_s)\bar{B}^T\right\}$$
$$\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\bar{A}_\tau e_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$
$$\mathbb{E}\{\Delta B_sf(e_s+\hat{x}_s)e_{s-\tau}^T\bar{A}_\tau^T+\bar{A}_\tau e_{s-\tau}f^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{\Delta A_se_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_se_{s-\tau}^T\bar{A}_\tau^T+\bar{A}_\tau e_{s-\tau}\varepsilon_s^TK_s^T\}\le\mathbb{E}\{\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}\bar{f}^T(e_s)\bar{B}^T+\bar{B}\bar{f}(e_s)e_{s-\tau}^T\Delta A_{d\tau}^T\}\le\mathbb{E}\{\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T+\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_{s-\tau}\bar{f}^T(e_s)\bar{B}^T+\bar{B}\bar{f}(e_s)\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$
$$\mathbb{E}\{\Delta B_sf(e_s+\hat{x}_s)\bar{f}^T(e_s)\bar{B}^T+\bar{B}\bar{f}(e_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}\le\mathbb{E}\{\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T+\Delta B_sf(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_s\bar{f}^T(e_s)\bar{B}^T+\bar{B}\bar{f}(e_s)\varepsilon_s^TK_s^T\}\le\mathbb{E}\{\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T+K_s\varepsilon_s\varepsilon_s^TK_s^T\}$$
$$\mathbb{E}\{\Delta A_s\hat{x}_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+\Delta A_{d\tau}e_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}\le\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+\Delta A_s\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_s^T\}$$

$$\mathbb{E}\{\Delta B_s f(e_s+\hat{x}_s)e_{s-\tau}^T\Delta A_{d\tau}^T+\Delta B_s f(e_s+\hat{x}_s)e_{s-\tau}^T\Delta A_{d\tau}^T\}\leq\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+\Delta B_s f(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_s e_{s-\tau}^T\Delta A_{d\tau}^T+\Delta A_{d\tau}e_{s-\tau}\varepsilon_s^T K_s^T\}\leq\mathbb{E}\{\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+K_s\varepsilon_s\varepsilon_s^T K_s^T\}$$
$$\mathbb{E}\{\Delta B_s f(e_s+\hat{x}_s)\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+\Delta A_{d\tau}\hat{x}_{s-\tau}f^T(e_s+\hat{x}_s)\Delta B_s^T\}\leq\mathbb{E}\{\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+\Delta B_s f(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_s\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+\Delta A_{d\tau}\hat{x}_{s-\tau}\varepsilon_s^T K_s^T\}\leq\mathbb{E}\{\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+K_s\varepsilon_s\varepsilon_s^T K_s^T\}$$
$$\mathbb{E}\{K_s\varepsilon_s f^T(e_s+\hat{x}_s)\Delta B_s^T+\Delta B_s f(e_s+\hat{x}_s)\varepsilon_s^T K_s^T\}\leq\mathbb{E}\{\Delta B_s f(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\Delta B_s^T+K_s\varepsilon_s\varepsilon_s^T K_s^T\}$$
$$\mathbb{E}\{-K_s\varepsilon_s v_{2s}^T E_s^T K_s^T-K_s E_s v_{2s}\varepsilon_s^T K_s^T\}\leq\mathbb{E}\{K_s E_s v_{2s}v_{2s}^T E_s^T K_s^T+K_s\varepsilon_s\varepsilon_s^T K_s^T\}.$$

Through collation and integration, the following results are obtained:

$$\begin{aligned}X_{s+1}\leq\mathbb{E}\{&10\bar{A}e_s e_s^T\bar{A}^T+10K_sD_se_se_s^TD_s^TK_s^T+10\Delta A_se_se_s^T\Delta A_s^T+10\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+10\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\\&+10\bar{B}\bar{f}(e_s)\bar{f}^T(e_s)\bar{B}^T+10\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+10\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+10\Delta B_sf(e_s+\hat{x}_s)\\&\times f^T(e_s+\hat{x}_s)\Delta B_s^T+11K_s\varepsilon_s\varepsilon_s^TK_s^T+C_sv_{1s}v_{1s}^TC_s^T+2K_sE_sv_{2s}v_{2s}^TE_s^TK_s^T\}\end{aligned}$$

It follows from Lemma 2 that

$$\mathbb{E}\{\bar{f}(e_s)\bar{f}^T(e_s)\}\leq\mathbb{E}\left\{tr\left(\bar{f}(e_s)\bar{f}^T(e_s)\right)\right\}I\leq\mathfrak{X}\mathbb{E}\{e_s^Te_s\}I$$
$$\begin{aligned}\mathbb{E}\{f(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s)\}&\leq\mathbb{E}\{tr(f(e_s+\hat{x}_s)f^T(e_s+\hat{x}_s))\}I\\&\leq2\mathfrak{X}\mathbb{E}\{e_s^Te_s\}I+2\mathfrak{X}\mathbb{E}\{\hat{x}_s^T\hat{x}_s\}I\end{aligned}$$

where $\mathfrak{X}$ is defined in (27). Furthermore, it is obvious to obtain

$$\begin{aligned}X_{s+1}\leq\mathbb{E}\{&10\bar{A}e_s e_s^T\bar{A}^T+10K_sD_se_se_s^TD_s^TK_s^T+10\Delta A_se_se_s^T\Delta A_s^T+10\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+10\bar{A}_\tau e_{s-\tau}e_{s-\tau}^T\bar{A}_\tau^T\\&+10\mathfrak{X}\bar{B}e_s^Te_s\bar{B}^T+10\Delta A_{d\tau}e_{s-\tau}e_{s-\tau}^T\Delta A_{d\tau}^T+10\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+20\mathfrak{X}\Delta B_se_s^Te_s\Delta B_s^T+20\mathfrak{X}\Delta B_s\\&\times\hat{x}_s^T\hat{x}_s\Delta B_s^T+11K_s\varepsilon_s\varepsilon_s^TK_s^T+C_sv_{1s}v_{1s}^TC_s^T+2K_sE_sv_{2s}v_{2s}^TE_s^TK_s^T\}\end{aligned}\tag{29}$$

Noting the fact

$$\varepsilon_s\varepsilon_s^T\leq\varepsilon_s^T\varepsilon_sI$$

we have

$$\mathbb{E}\{\varepsilon_s^T\varepsilon_s\}I\leq\varpi I$$

where ϖ is defined in (27). According to the property of the trace, it is straightforward to get

$$\begin{aligned}\mathbb{E}\{e_s^Te_s\}&=\mathbb{E}\{tr(e_se_s^T)\}=\mathrm{tr}(X_s)\\\hat{x}_s^T\hat{x}_s&=\mathrm{tr}(\hat{x}_s\hat{x}_s^T)\end{aligned}\tag{30}$$

Combining (29) with (30) results in

$$\begin{aligned}X_{s+1}\leq\ &10\bar{A}X_s\bar{A}^T+10K_sD_sX_sD_s^TK_s^T+10\Delta A_sX_s\Delta A_s^T+10\Delta A_s\hat{x}_s\hat{x}_s^T\Delta A_s^T+10\bar{A}_\tau X_{s-\tau}\bar{A}_\tau^T\\&+10\mathfrak{X}\mathrm{tr}(X_s)\bar{B}\bar{B}^T+10\Delta A_{d\tau}X_{s-\tau}\Delta A_{d\tau}^T+10\Delta A_{d\tau}\hat{x}_{s-\tau}\hat{x}_{s-\tau}^T\Delta A_{d\tau}^T+20\mathfrak{X}\mathrm{tr}(X_s)\Delta B_s\Delta B_s^T\\&+20\mathfrak{X}\mathrm{tr}(\hat{x}_s\hat{x}_s^T)\Delta B_s\Delta B_s^T+11\varpi K_sK_s^T+C_s\mathcal{V}_{1s}C_s^T+2K_sE_s\mathcal{V}_{2s}E_s^TK_s^T\\=\ &\mathfrak{T}(X_s)\end{aligned}$$

Noticing $\mathcal{G}_0\geq X_0$ and letting $\mathcal{G}_s\geq X_s$, one has

$$\mathfrak{T}(\mathcal{G}_s)\geq\mathfrak{T}(X_s)\geq X_{s+1}\tag{31}$$

From (26) and (31), we obtain

$$\mathcal{G}_{s+1} \geq \mathfrak{T}(\mathcal{G}_s) \geq \mathfrak{T}(X_s) \geq X_{s+1} \quad (32)$$

Therefore, the proof is now complete.

Based on the analysis of Theorem 1 and Theorem 2 mentioned above, the following sufficient criteria are obtained, which can guarantee the two desired constraints.

Theorem 3: Consider the MNNs with variance constraint (1). Assume that the TVSE gain matrix K_s in (7) is given. For given scalar $\gamma > 0$, PDRVMs $\mathcal{U}_\varphi$, $\mathcal{U}_\phi$ and $\mathcal{U}_\psi$, under the initial conditions $\mathcal{G}_0 = X_0$, $\mathcal{R}_0 \leq \gamma^2 \mathcal{U}_\phi$ and $\mathfrak{Q}_l \leq \gamma^2 \mathcal{U}_\psi (l = -\tau, -\tau+1, \dots, -1)$, if there exist PDRVMs $\{\mathcal{R}_s\}_{1\leq s\leq N+1}$, $\{\mathcal{G}_s\}_{1\leq s\leq N+1}$ and $\{\mathfrak{Q}_s\}_{0\leq s\leq N}$ satisfying the inequalities

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Sigma_{15} & 0 & 0 & 0 & 0 \\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & \Sigma_{26} & \Sigma_{27} & \Sigma_{28} & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & 0 & \Sigma_{38} & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & \Sigma_{49} \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{99} \end{bmatrix} < 0 \quad (33)$$

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ * & \Upsilon_{22} & 0 & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44} \end{bmatrix} < 0 \quad (34)$$

where

$$\Sigma_{11} = -\mathcal{R}_s + \mathfrak{Q}_s - R_{1s} + M_s^T M_s, \Sigma_{12} = [-R_{2s} \quad -R_{3s}^T]$$

$$\Sigma_{14} = [0 \quad 0 \quad \bar{A}^T], \Sigma_{15} = \begin{bmatrix} 3\bar{A}^T & \sqrt{11} D_s^T K_s^T & \sqrt{11} \Delta A_s^T \end{bmatrix}$$

$$\Sigma_{22} = \begin{bmatrix} -I & f(\hat{x}_s) \\ * & -f^T(\hat{x}_s) f(\hat{x}_s) + \dfrac{\sigma}{\theta} - \kappa_s \sigma \end{bmatrix}, \Sigma_{24} = \begin{bmatrix} 0 & 0 & \bar{B}^T \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Sigma_{26} = \begin{bmatrix} \sqrt{11} \Delta B_s^T & 3\bar{B}^T \\ 0 & 0 \end{bmatrix}, \Sigma_{27} = \begin{bmatrix} 0 & 0 \\ \sqrt{11} \hat{x}_s^T \Delta A_s^T & \sqrt{11} f^T(\hat{x}_s) \bar{B}^T \end{bmatrix}$$

$$\Sigma_{28} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{11} \hat{x}_{s-\tau}^T \Delta A_{d\tau}^T & 0 & 0 & 0 \end{bmatrix}, \Sigma_{33} = \text{diag}\left\{-\mathfrak{Q}_{s-\tau}, -\left(\frac{1}{\theta} - \kappa_s\right) I, \frac{\lambda - 1 + \kappa_s}{\theta} I\right\}$$

$$\Sigma_{38} = \begin{bmatrix} 0 & \sqrt{11} \bar{A}_\tau^T & \sqrt{11} \Delta A_{d\tau}^T & 0 \\ 0 & 0 & 0 & 2\sqrt{3} K_s^T \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Sigma_{49} = \begin{bmatrix} C_s^T & 0 \\ 0 & \sqrt{2} E_s^T K_s^T \\ 0 & 0 \end{bmatrix}$$

$$\Sigma_{44} = \text{diag}\{-\gamma^2 \mathcal{U}_\varphi, -\gamma^2 \mathcal{U}_\varphi, -\mathcal{R}_{s+1}^{-1}\}, \Sigma_{55} = \text{diag}\{-\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}\}$$

$$\Sigma_{66} = \text{diag}\{-\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}\}, \Sigma_{77} = \text{diag}\{-\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}\}$$

$$\Sigma_{88} = \text{diag}\{-\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}\}, \Sigma_{99} = \text{diag}\{-\mathcal{R}_{s+1}^{-1}, -\mathcal{R}_{s+1}^{-1}\}$$

$$\Upsilon_{11} = -\mathcal{G}_{s+1} + 10\bar{A}\mathcal{G}_s\bar{A}^T + 10\bar{A}_\tau \mathcal{G}_{s-\tau} \bar{A}_\tau^T + 10\mathfrak{X}\text{tr}(\mathcal{G}_s)\bar{B}\bar{B}^T + C_s \mathcal{V}_{1s} C_s^T$$

$$\Upsilon_{12} = \begin{bmatrix} \sqrt{10} K_s D_s \mathcal{G}_s & \sqrt{10} \Delta A_s \mathcal{G}_s & \sqrt{10} \Delta A_s \hat{x}_s \end{bmatrix}$$

$$\Upsilon_{13} = \begin{bmatrix} \sqrt{11\varpi} K_s & \sqrt{10} \Delta A_{d\tau} \mathcal{G}_{s-\tau} & \sqrt{10} \Delta A_{d\tau} \hat{x}_{s-\tau} \end{bmatrix}$$

$$\Upsilon_{14} = \begin{bmatrix} 2\sqrt{5\mathfrak{X}}\text{tr}(\mathcal{G}_s)\Delta B_s & 2\sqrt{5\mathfrak{X}}\text{tr}(\hat{x}_s \hat{x}_s^T)\Delta B_s & \sqrt{2} K_s E_s \mathcal{V}_{2s} \end{bmatrix}$$

$$\Upsilon_{22} = \text{diag}\{-\mathcal{G}_s, -\mathcal{G}_s, -I\}, \Upsilon_{33} = \text{diag}\{-I, -\mathcal{G}_{s-\tau}, -I\}$$

$$\Upsilon_{44} = \text{diag}\{-\text{tr}(\mathcal{G}_s) I, -\text{tr}(\hat{x}_s \hat{x}_s^T) I, -\mathcal{V}_{2s}\}$$

then two desirable constraints can be satisfied simultaneously.

Proof: In terms of Theorem 1 and Theorem 2, the H_∞ performance constraint in (10) obeys $J_1 > 0$ and the EVB obeys $J_2 \coloneqq X_s \leq \mathfrak{Y}_s$, that is, the inequality (33) implies (16) and (34) implies (26) under the initial conditions, and we obtain sufficient conditions to ensure the desired H_∞ performance constraint and the EVB, which completes the proof of Theorem 3.

In the end, the following theorem is obtained to provide a solvable algorithm for the TVSE gain.

Theorem 4: Consider the MNNs (1) under variance constraint. For given the attenuation level $\gamma > 0$, the PDRVMs $\mathcal{U}_\phi$, $\mathcal{U}_\varphi$ and $\mathcal{U}_\psi$, a series of matrices $\{\mathfrak{Y}_s\}_{0\leq s\leq N+1}$, under the initial condition

$$\begin{cases}\mathcal{R}_0 \leq \gamma^2 \mathcal{U}_\phi \\ \mathfrak{Q}_l \leq \gamma^2 \mathcal{U}_\psi, \ (l = -\tau, -\tau+1, \dots, -1) \\ \mathbb{E}\{e_0 e_0^T\} = \mathcal{G}_0 \leq \mathfrak{Y}_0\end{cases} \tag{35}$$

if there exist PDRVMs $\{\mathcal{R}_s\}_{1\leq s\leq N+1}$, $\{\mathcal{G}_s\}_{1\leq s\leq N+1}$ and $\{\mathfrak{Q}_s\}_{0\leq s\leq N}$, the estimator gain matrix $\{K_s\}_{0\leq s\leq N}$ and scalars $\epsilon_{i,s} > 0 (i = 1,2, \dots ,8)$ obeying the inequalities

$$\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & 0 \\ * & * & \Omega_{33}\end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix}\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & 0 & 0 & 0 \\ * & \Upsilon_{22} & 0 & 0 & \mathcal{W}_s^T & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & \mathcal{X}_s^T & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 & \mathcal{Y}_s^T \\ * & * & * & * & -\epsilon_{6,s} I & 0 & 0 \\ * & * & * & * & * & -\epsilon_{7,s} I & 0 \\ * & * & * & * & * & * & -\epsilon_{8,s} I\end{bmatrix} < 0 \tag{37}$$

$$\mathcal{G}_{s+1} - \mathfrak{Y}_{s+1} \leq 0 \tag{38}$$

with the following updating rule

$$\overline{\mathcal{R}}_s = \mathcal{R}_s^{-1} \tag{39}$$

where

$$\Omega_{11} = \begin{bmatrix}\Xi_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Xi_{15} & 0 & 0 & 0 & 0 \\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & \Xi_{26} & \Xi_{27} & 0 & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & 0 & \Xi_{38} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & \Sigma_{49} \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99}\end{bmatrix}$$

$$\Omega_{12} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon_{2,s}\mathcal{N}_2 & 0 & \epsilon_{3,s}\mathcal{N}_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \mathcal{H}_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{H}_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{H}_3^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix}, \Omega_{13} = \begin{bmatrix}0 & 0 & 0 & 0 \\ \epsilon_{4,s}\mathcal{N}_4 & 0 & 0 & 0 \\ 0 & 0 & \epsilon_{5,s}\mathcal{N}_5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathcal{H}_4^T & 0 & \mathcal{H}_5^T \\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Omega_{22} = \text{diag}\{-\epsilon_{1,s}I, -\epsilon_{2,s}I, -\epsilon_{2,s}I, -\epsilon_{3,s}I, -\epsilon_{3,s}I\}, \Omega_{33} = \text{diag}\{-\epsilon_{4,s}I, -\epsilon_{4,s}I, -\epsilon_{5,s}I, -\epsilon_{5,s}I\}$$

$$\Xi_{11} = -\mathcal{R}_s + \mathfrak{Q}_s - R_{1s} + M_s^T M_s + \epsilon_{1,s}N_1^T N_1, \Sigma_{12} = [-R_{2s} \quad -R_{3s}^T]$$

$$\Sigma_{14} = [0 \quad 0 \quad \bar{A}^T], \Xi_{15} = \begin{bmatrix}3\bar{A}^T & \sqrt{11}D_s^T K_s^T & 0\end{bmatrix}$$

$$\Sigma_{24} = \begin{bmatrix} 0 & 0 & \bar{B}^T \\ 0 & 0 & 0 \end{bmatrix}, \Xi_{26} = \begin{bmatrix} 0 & 3\bar{B}^T \\ 0 & 0 \end{bmatrix}$$

$$\Xi_{27} = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{11}f^T(\hat{x}_s)\bar{B}^T \end{bmatrix}, \Sigma_{22} = \begin{bmatrix} -I & f(\hat{x}_s) \\ * & -f^T(\hat{x}_s)f(\hat{x}_s) + \frac{\sigma}{\theta} - \kappa_s\sigma \end{bmatrix}$$

$$\Sigma_{33} = \text{diag}\left\{-\mathfrak{Q}_{s-\tau}, -\left(\frac{1}{\theta} - \kappa_s\right)I, \frac{\lambda - 1 + \kappa_s}{\theta}I\right\}$$

$$\Xi_{38} = \begin{bmatrix} 0 & \sqrt{11}\bar{A}_\tau^T & 0 & 0 \\ 0 & 0 & 0 & 2\sqrt{3}K_s^T \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Sigma_{49} = \begin{bmatrix} C_s^T & 0 \\ 0 & \sqrt{2}E_s^T K_s^T \\ 0 & 0 \end{bmatrix}$$

$$\Xi_{44} = \text{diag}\{-\gamma^2\mathcal{U}_\varphi, -\gamma^2\mathcal{U}_\varphi, -\overline{\mathcal{R}}_{s+1}\}, \Xi_{55} = \text{diag}\{-\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}\}$$

$$\Xi_{66} = \text{diag}\{-\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}\}, \Xi_{77} = \text{diag}\{-\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}\}$$

$$\Xi_{88} = \text{diag}\{-\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}\}, \Xi_{99} = \text{diag}\{-\overline{\mathcal{R}}_{s+1}, -\overline{\mathcal{R}}_{s+1}\}$$

$$\Psi_{11} = -\mathcal{G}_{s+1} + 10\bar{A}\mathcal{G}_s\bar{A}^T + 10\bar{A}_\tau\mathcal{G}_{s-\tau}\bar{A}_\tau^T + 10\mathfrak{X}\text{tr}(\mathcal{G}_s)\bar{B}\bar{B}^T + C_s\mathcal{V}_{1s}C_s^T + (\epsilon_{6,s} + \epsilon_{7,s} + \epsilon_{8,s})HH^T$$

$$\Psi_{12} = \begin{bmatrix}\sqrt{10}K_s D_s \mathcal{G}_s & 0 & 0\end{bmatrix}, \Psi_{13} = \begin{bmatrix}\sqrt{11\varpi}K_s & 0 & 0\end{bmatrix}, \Psi_{14} = \begin{bmatrix}0 & 0 & \sqrt{2}K_s E_s \mathcal{V}_{2s}\end{bmatrix}$$

$$\Upsilon_{22} = \text{diag}\{-\mathcal{G}_s, -\mathcal{G}_s, -I\}, \Upsilon_{33} = \text{diag}\{-I, -\mathcal{G}_{s-\tau}, -I\}, \Upsilon_{44} = \text{diag}\{-\text{tr}(\mathcal{G}_s)I, -\text{tr}(\hat{x}_s\hat{x}_s^T)I, -\mathcal{V}_{2s}\}$$

$$\mathcal{H}_1 = [0 \quad 0 \quad \sqrt{11}H^T], \mathcal{N}_2^T = [N_3 \quad 0], \mathcal{H}_2 = [\sqrt{11}H^T \quad 0]$$

$$\mathcal{N}_3^T = [0 \quad N_1\hat{x}_s], \mathcal{H}_3 = [\sqrt{11}H^T \quad 0], \mathcal{N}_4^T = [0 \quad N_2\hat{x}_{s-\tau}]$$

$$\mathcal{H}_4 = [\sqrt{11}H^T \quad 0 \quad 0 \quad 0], \mathcal{N}_5^T = [N_2 \quad 0 \quad 0], \mathcal{H}_5 = [0 \quad 0 \quad \sqrt{11}H^T \quad 0]$$

$$\mathcal{W}_s = \begin{bmatrix}0 & \sqrt{10}N_1\mathcal{G}_s & \sqrt{10}N_1\hat{x}_s\end{bmatrix}, \mathcal{X}_s = \begin{bmatrix}0 & \sqrt{10}N_2\mathcal{G}_{s-\tau} & \sqrt{10}N_2\hat{x}_{s-\tau}\end{bmatrix}$$

$$\mathcal{Y}_s = \begin{bmatrix}2\sqrt{5\mathfrak{X}}\text{tr}(\mathcal{G}_s)N_3 & 2\sqrt{5\mathfrak{X}}\text{tr}(\hat{x}_s\hat{x}_s^T)N_3 & 0\end{bmatrix}$$

then the TVSE gain can be obtained by solving the RLMIs (36)-(38).

Proof: Firstly, we handle the parameter uncertainty, (33) can be rewritten as

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Xi_{15} & 0 & 0 & 0 & 0 \\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & \Xi_{26} & \Xi_{27} & \Xi_{28} & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & 0 & \Xi_{38} & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & \Sigma_{49} \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{99} \end{bmatrix} + \bar{N}_s F_{1,s}^T \bar{H}_s + \bar{H}_s^T F_{1,s} \bar{N}_s^T + \hat{N}_s F_{3,s}^T \hat{H}_s + \hat{H}_s^T F_{3,s} \hat{N}_s^T + \tilde{N}_s F_{1,s}^T \tilde{H}_s$$
$$+ \tilde{H}_s^T F_{1,s} \tilde{N}_s^T + \vec{N}_s F_{2,s}^T \vec{H}_s + \vec{H}_s^T F_{2,s} \vec{N}_s^T + \breve{N}_s F_{2,s}^T \breve{H}_s + \breve{H}_s^T F_{2,s} \breve{N}_s^T < 0$$

where

$$\bar{N}_s^T = [N_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \bar{H}_s = [0 \quad 0 \quad 0 \quad 0 \quad \mathcal{H}_1 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\hat{N}_s^T = [0 \quad \mathcal{N}_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \hat{H}_s = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \mathcal{H}_2 \quad 0 \quad 0 \quad 0]$$

$$\tilde{N}_s^T = [0 \quad \mathcal{N}_3^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \tilde{H}_s = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \mathcal{H}_3 \quad 0 \quad 0]$$

$$\vec{N}_s^T = [0 \quad \mathcal{N}_4^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \vec{H}_s = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \mathcal{H}_4 \quad 0]$$

$$\breve{N}_s^T = [0 \quad 0 \quad \mathcal{N}_5^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \breve{H}_s = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \mathcal{H}_5 \quad 0]$$

Furthermore, it is not difficult to derive that

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Xi_{15} & 0 & 0 & 0 & 0 \\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & \Xi_{26} & \Xi_{27} & \Xi_{28} & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & 0 & \Xi_{38} & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & \Sigma_{49} \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{99} \end{bmatrix} + \epsilon_{1,s}\bar{N}_s\bar{N}_s^T + \epsilon_{1,s}^{-1}\bar{H}_s^T\bar{H}_s + \epsilon_{2,s}\hat{N}_s\hat{N}_s^T + \epsilon_{2,s}^{-1}\hat{H}_s^T\hat{H}_s + \epsilon_{3,s}\tilde{N}_s\tilde{N}_s^T + \epsilon_{3,s}^{-1}\tilde{H}_s^T\tilde{H}_s$$
$$+\epsilon_{4,s}\vec{N}_s\vec{N}_s^T + \epsilon_{4,s}^{-1}\vec{H}_s^T\vec{H}_s + \epsilon_{5,s}\breve{N}_s\breve{N}_s^T + \epsilon_{5,s}^{-1}\breve{H}_s^T\breve{H}_s < 0$$

Similarly, based on (34), we can get

$$\begin{bmatrix} \Upsilon_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ * & \Upsilon_{22} & 0 & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44} \end{bmatrix} + \mathfrak{N}_s F_{1,s}\mathfrak{H}_{1,s} + \mathfrak{H}_{1,s}^T F_{1,s}^T \mathfrak{N}_s^T + \mathfrak{N}_s F_{2,s}\mathfrak{H}_{2,s}$$
$$+\mathfrak{H}_{2,s}^T F_{2,s}^T \mathfrak{N}_s^T + \mathfrak{N}_s F_{3,s}\mathfrak{H}_{3,s} + \mathfrak{H}_{3,s}^T F_{3,s}^T \mathfrak{N}_s^T < 0$$

where

$$\mathfrak{N}_s^T = [H^T \quad 0 \quad 0 \quad 0]$$
$$\mathfrak{H}_{1,s} = [0 \quad \mathcal{W}_s \quad 0 \quad 0]$$
$$\mathfrak{H}_{2,s} = [0 \quad 0 \quad \mathcal{X}_s \quad 0]$$
$$\mathfrak{H}_{3,s} = [0 \quad 0 \quad 0 \quad \mathcal{Y}_s]$$

Furthermore, we obtain

$$\begin{bmatrix} \Upsilon_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ * & \Upsilon_{22} & 0 & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44} \end{bmatrix} + \epsilon_{6,s}\mathfrak{N}_s\mathfrak{N}_s^T + \epsilon_{6,s}^{-1}\mathfrak{H}_{1,s}^T\mathfrak{H}_{1,s} + \epsilon_{7,s}\mathfrak{N}_s\mathfrak{N}_s^T + \epsilon_{7,s}^{-1}\mathfrak{H}_{2,s}^T\mathfrak{H}_{2,s} + \epsilon_{8,s}\mathfrak{N}_s\mathfrak{N}_s^T + \epsilon_{8,s}^{-1}\mathfrak{H}_{3,s}^T\mathfrak{H}_{3,s} < 0$$

Thus, we can conclude that two desired requirements can be achieved simultaneously.

Remark 2: Contrary to existing approaches, the key distinctions of the proposed H_∞ SE scheme are listed as follows: (1) the developed H_∞ SE method aims to ensure that the error covariance is bounded, which can be adapted to fulfill the admissibility of the presented H_∞ SE strategy to some extent; and (2) the disturbance attenuation level can be achieved, where the sufficient condition is derived to ensure the desired H_∞ performance constraint within the time-varying framework. Consequently, the proposed H_∞ SE scheme obeys both the predefined H_∞ performance requirement and the EVB, which might provide more application domains.

Remark 3: Inequalities (33) and (34) in Theorem 3 are mainly derived from the results of Theorem 1 and Theorem 2 via the Schur complement lemma. The reason lies in that we aim to solve the gain matrices of the finite-horizon estimator, thus it is necessary to handle the nonlinear terms, which also requires using the Schur complement lemma to convert the recursive matrix inequalities into symmetric block matrix inequalities. Inequalities (36) and (38) in Theorem 4 are obtained by addressing the uncertainties in the two matrix inequalities of Theorem 3 through the S-procedure and expressing $\mathcal{R}_s^{-1}$ (the inverse of $\overline{\mathcal{R}}_s$) using updated matrices, and the proof section of Theorem 4 focuses on handling these uncertainties.

4. A Simulation Example

In this section, the main purpose is to demonstrate the effectiveness of newly proposed H_∞ SE method under variance constraint.

For the MNNs (1), the related parameters are given as follows:

$$a_1(x_{1,s})=\begin{cases}-0.66, |x_{1,s}|>1\\0.22, |x_{1,s}|\le 1\end{cases}, a_2(x_{2,s})=\begin{cases}-0.45, |x_{2,s}|>1\\-0.15, |x_{2,s}|\le 1\end{cases}, a_{11,\tau}(x_{1,s})=\begin{cases}-0.53, |x_{1,s}|>1\\0.29, |x_{1,s}|\le 1\end{cases}$$

$$a_{12,\tau}(x_{1,s})=\begin{cases}0.31, |x_{1,s}|>1\\0.11, |x_{1,s}|\le 1\end{cases}, a_{21,\tau}(x_{2,s})=\begin{cases}-0.43, |x_{2,s}|>1\\0.21, |x_{2,s}|\le 1\end{cases}, a_{22,\tau}(x_{2,s})=\begin{cases}-0.54, |x_{2,s}|>1\\0.32, |x_{2,s}|\le 1\end{cases}$$

$$b_{11}(x_{1,s})=\begin{cases}-0.54, |x_{1,s}|>1\\0.12, |x_{1,s}|\le 1\end{cases}, b_{12}(x_{1,s})=\begin{cases}0.41, |x_{1,s}|>1\\-0.17, |x_{1,s}|\le 1\end{cases}, b_{21}(x_{2,s})=\begin{cases}-0.33, |x_{2,s}|>1\\0.11, |x_{2,s}|\le 1\end{cases}$$

$$b_{22}(x_{2,s})=\begin{cases}-0.21, |x_{2,s}|>1\\-0.23, |x_{2,s}|\le 1\end{cases}, C_s=[-0.31 \quad -0.15\, sin(2s)]^T, \quad E_s=[-0.22 \quad -0.15\, sin(2s)]^T$$

$$D_s=\begin{bmatrix}0.11\, sin(2s) & -0.18\\ 0.21 & -0.21\, sin(2s)\end{bmatrix}, M_s=[-0.03 \quad -0.21\, sin(3s)]$$

$$\rho=0.7, \quad \tau=2, \quad \sigma=0.7, \quad \lambda=1.8$$

It is straightforward to derive that

$$H=\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix}, \quad N_1=\begin{bmatrix}0.44 & 0\\0 & 1\\1 & 0\\0 & 0.15\end{bmatrix}$$

$$N_2=\begin{bmatrix}0.41 & 0\\0 & 0.1\\0.32 & 0\\0 & 0.43\end{bmatrix}, \quad N_3=\begin{bmatrix}0.33 & 0\\0 & 0.29\\0.22 & 0\\0 & 0.01\end{bmatrix}$$

Additionally, choose the activation function as follows:

$$f(x_s)=\begin{bmatrix}0.48x_{1,s}+\tanh(0.08x_{1,s})+0.16x_{2,s}\\0.37x_{1,s}+0.3x_{2,s}+\tanh(0.06x_{2,s})\end{bmatrix}$$

where the state variable is represented as $x_s=[x_{1,s} \quad x_{2,s}]^T$, the mean of the initial value is $\phi_0=[2.4 \quad 0.7]^T$, the initial value is $\hat{x}_0=[-0.2 \quad 0.5]^T$ and initial values of time-delay are $\phi_{-1}=[-1 \quad 2]^T$, $\phi_{-2}=[2 \quad -1]^T$, $\hat{x}_{-1}=[1 \quad -1]^T$ and $\hat{x}_{-2}=[1 \quad -2]^T$. Set the weighted matrices $\mathcal{U}_\varphi=I$, $\mathcal{U}_\phi=0.1I$ and $\mathcal{U}_\psi=I$, the correlation matrices of the activation function are $\boldsymbol{U}_1=\begin{bmatrix}0.56 & 0.37\\0.37 & 0.36\end{bmatrix}$ and $\boldsymbol{U}_2=\begin{bmatrix}0.48 & 0.16\\0.16 & 0.3\end{bmatrix}$, the variance upper bounds $\{\mathfrak{Y}_s\}_{0\le s\le N+1}=\mathrm{diag}\{0.3,0.3\}$, the attenuation level $\gamma=0.6$, covariances $\mathcal{V}_{1s}=\mathcal{V}_{2s}=1$ and $N=80$. For demonstration purposes, two cases are considered with different values: Case I: $\theta=0.6$; Case II: $\theta=6$. Furthermore, according to (36)-(38), the TVSE gain matrix K_s is designed in Table **2** and Table **3**.

Table 2: TVSE gain matrices (Case I: $\theta = 0.6$).

s	K_s
1	$K_1 = \begin{bmatrix} 0.2452 & -0.2542 \\ 0.1132 & 0.5611 \end{bmatrix}$
2	$K_2 = \begin{bmatrix} 0.3462 & -0.4526 \\ -0.1355 & 0.1355 \end{bmatrix}$
3	$K_3 = \begin{bmatrix} 0.3511 & 0.5461 \\ 0.1543 & 0.4514 \end{bmatrix}$
⋮	⋮

Table 3: TVSE gain matrices (Case II: $\theta = 6$).

s	K_s
1	$K_1 = \begin{bmatrix} -0.4262 & 0.3562 \\ 0.4352 & 0.2354 \end{bmatrix}$
2	$K_2 = \begin{bmatrix} 0.3451 & -0.2455 \\ 0.3515 & 0.4351 \end{bmatrix}$
3	$K_3 = \begin{bmatrix} 0.3241 & -0.4221 \\ 0.3452 & -0.5426 \end{bmatrix}$
⋮	⋮

In the simulation, the norm sum of the controlled output EEs can be calculated under two cases. From Table **3**, it is not difficult to conclude that the norm sum of the controlled output EEs in Case I is smaller than that in Case II. By comparing two cases, we can conclude that a larger θ value leads to more updated information, and the estimation accuracy is relatively better. In addition, the triggering rate L_s is defined as the transmission performance level by $L_s = \frac{\mathcal{N}_s}{N}$, where $\mathcal{N}_s$ stands for the number of actually transmitted data and $N = 80$ depicts the length of finite-horizon. Furthermore, in order to obtain the relationship between the triggering rate and parameter θ, the triggering rate is given in Table **4** with different values of θ. When θ increases, the trigger rate monotonically increases. θ is the threshold parameter of the DETM, and a larger θ results in looser triggering conditions, leading to increased frequencies of data transmission and computation execution, thereby reducing resource-saving effects. Therefore, a larger θ leads to a higher event-triggering rate and weaker resource-saving performance.

The sensitivity analysis and parameter tuning rationale are supplemented for parameters ρ, σ, λ and γ as follows: (1) $\rho \in (0,1)$ is a parameter for handling the sector-bounded condition of the nonlinear activation function. (2) $\sigma > 0$ and $\lambda > 0$ are core parameters of the DETM, controlling the triggering threshold and the update rate of the internal dynamic variable. (3) $\gamma > 0$ is the H_∞ performance index and represents the disturbance attenuation level of the system. We solved the RLMIs to obtain the γ satisfying $J_1 < 0$.

Table 4: Comparisons with EEs of controlled output.

	$\sum_{s=0}^{N-1} \|\tilde{z}_s\|^2$
Case I: $\theta = 0.6$	3.2423
Case II: $\theta = 6$	0.6745

The simulation results are given in Fig. (**2-5**). Fig. (**2**) describes the controlled output z_s and its estimation $\hat{z}_s$, and Fig. (**3**) depicts the EEs of controlled output$\tilde{z}_s$. Fig. (**4**) and Fig. (**5**) describe the actual error variance and upper bound of error variance e_s and the upper bound with different values of θ. It is observed that the upper bound of error variance decreases monotonically as θ increases. According to the Table **4** and Fig. (**5**), we can conclude that the presented DETM reduces the communication burden at the cost of sacrificing certain estimation performance of

MNNs. According to the above analysis, the simulation results demonstrate the effectiveness of the proposed H_∞ SE algorithm under DETMs.

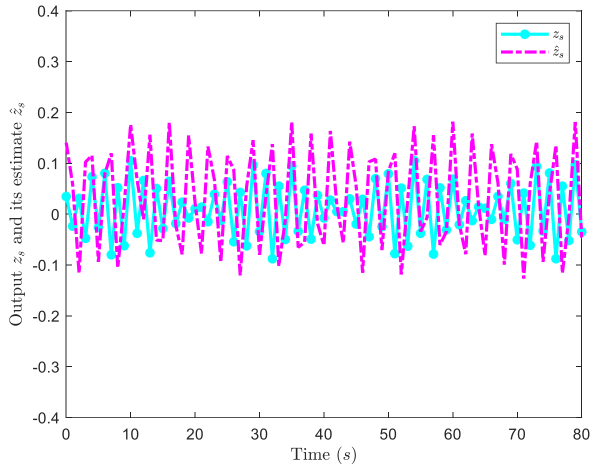

Figure 2: The controlled output z_s along with its estimated value.

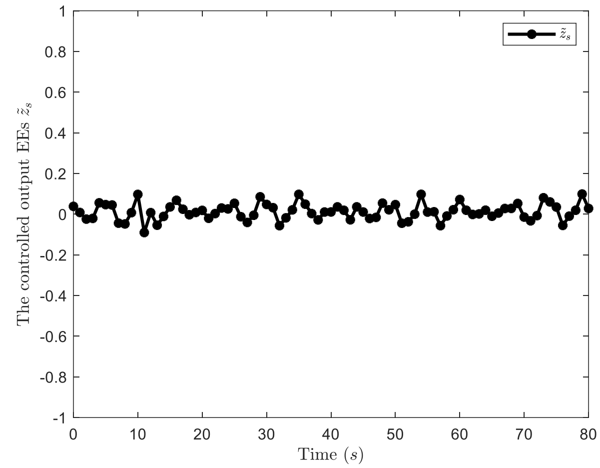

Figure 3: The EEs of controlled output $\tilde{z}_s$.

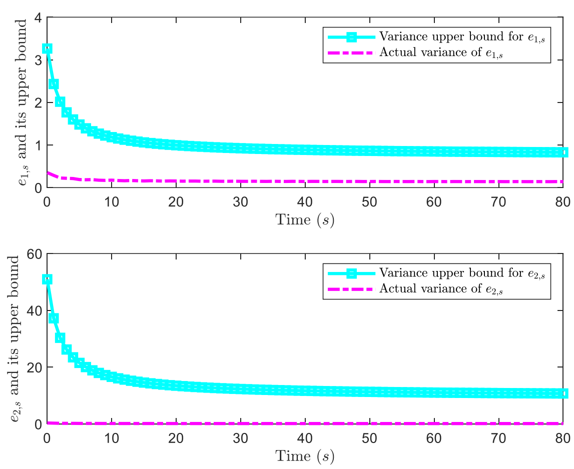

Figure 4: The actual error variance and upper bound of error variance e_s.

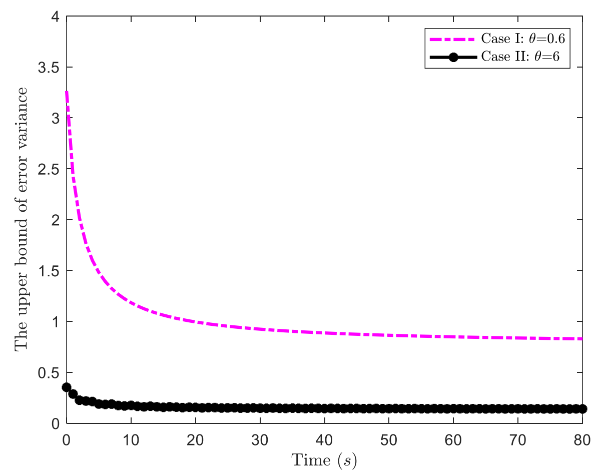

Figure 5: Upper bound trajectories for different θ.

5. Conclusion

This paper has tackled the dynamic event-triggered H_∞ SE problem for MNNs with time-delay under variance constraint. For the purpose of avoiding resource consumption in the communication channel, the DETM has been introduced into the sensor-to-estimator. The TVSE has been designed for MNNs with variance constraint and time-delay, where sufficient conditions have been obtained to guarantee two constraints including the specified H_∞ performance constraint and the EVB. Especially, a novel dynamic event-triggered H_∞ SE method has been proposed without using the augmentation algorithm, and the TVSE gain has been given via the RLMIs method and the stochastic analysis techniques. Finally, the effectiveness of the presented H_∞ SE method has been verified by a simulation example. This paper has discussed the dynamic event-triggered H_∞ state estimation for MNNs with variance constraints and time-delay, but there are still many topics worth studying. For example, other communication protocols can be adopted, such as the event-triggering round-robin-like protocol, the FlexRay communication protocol and so on, which we will consider in depth in the subsequent study.

Conflict of Interest

No potential conflict of interest was reported by the authors.

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