Quantum Game Techniques Applied to Wireless Networks Communications

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Abstract: In order to analyze the power control problem, the wireless quantum network nodes are modeled as players at a quantum game. The power control problem is one of the most significant wireless communications challenges which characteristics make it proper to be modeled by means of game theory techniques. The problem results in non-cooperative game by nature, but, under quantum rules, a larger strategy space leads the players to choose a coalition strategy as the best option. Thus, the use of quantum game strategies makes possible the emergence of new equilibrium, which guarantees the best possible performance to the whole network. We show also that the whole network power consumption decreases when the intrinsic parallel behavior of quantum computation is capitalized. Moreover, the design of efficient medium access control algorithms is possible.

Keywords: Game theory, Quantum Computation, Wireless Communications.

1. INTRODUCTION

In the last decades, there has been a breakthrough in wireless communication networks. Many types of portable communication devices, such as smartphones, tablets, PDAs are carried by many people for use in the different domains of their lives. Thus, given the plenty of transmission protocols and software radio capabilities, networks are evolving to less structured and increasingly involve distributed decision making. Power control problems games are about the right amount of power the nodes of a network must use to send information through the available channels. When the used power increases, the wireless devices battery life diminishes and the interference between users increases. On the other hand, there is a minimum of transmission power that satisfies the quality of service thresholds.

In other words, from a particular user point of view, an efficient power control algorithm must support him with some minimum acceptable throughput, whereas from the whole network point of view, the aggregate throughput must be maximized. Accordingly, many techniques have proven to be successful by various authors and researchers for this purpose. Among these techniques, because of the problem characteristics, the most appropriate are based on game theory models.

The application of game theory dates back to the 90's. Game theory is the study of strategic decision making, where the decision makers are players whose utilities (or payoffs) depend on other players' actions [1, 2]. Particularly, in a multiuser wireless network system the nodes behave like players in a game [3-9], so that they compete or even cooperate in order to achieve the wanted quality of service.

In [10] for instance, the authors studied the competitive and cooperative distributed spectrum coordination techniques for the two users Gaussian interference game. The author shows that the most used IWF (Iterative water-filling) algorithm is not optimum. The IWF algorithm converges to a situation in which the power of one transmitter is allocated uniformly in every possible channel [11, 12], however a problem arises when more than one transmitter are involved. Because of the competing interests, this situation can lead to the prisoner's dilemma [13]. The prisoner's dilemma is about two persons who are arrested and put in separated rooms to be interrogated. The police talk to them and tell their options: If they both cooperate with each other (do not confess) they receive a minimum sentence, three months for example. If only one of them betrays (confess) this one is freed but the other is considered guilty of all the charges and given the maximum penalty, five years for example. On the other hand, if both betray and plead guilty, they receive a sentence of 1 year in jail. Thus, each prisoner must decide between to cooperate which would benefit both, but at risk of being betrayed or to betray in order to protect himself. As both of them receive the same deal, both decide to betray although through cooperation they could serve less prison time. Therefore, the problem has a Nash equilibrium, which is not Pareto optimum, i.e., users do not achieve the maximum rate. On the contrary, the quantum version of

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the prisoner's dilemma game does have an optimum solution [14]. In this framework, the quantum model presented here is game based model where players take their decisions thinking about not only but also on the others benefits in order that the whole network gets the best possible performance. In this way, quantum games larger space of strategies gives the players new chances inducing the system to new stability points.

In this paper, we present an *N* players interference quantum model and analyze players' performance of using classic or quantum strategies. Because of the network users must decide between the whole network health and their own, the prisoner's dilemma is also present in this case. However, the dilemma can be eliminated by means of quantum entanglement and quantum superposition, two features only feasible under quantum computation. Then, it is possible to consider a quantum phase, interleaved in the real classic protocols, that manages fairly the users' power spectrum.

2. A QUANTUM GAME OF INTERFERENCE

Power control becomes necessary when a set of wireless mobiles share a common network. The purpose is to let every user to send information without causing unnecessary interference to the others. That is, the most power they use, the harder the interference they can cause to the neighbors receptions. Besides, the more power they save, the longer is the wireless devices battery life. On the other hand, because there is a constraint in the minimum of transmission power necessary to satisfy the quality of service thresholds, the users can not reduce the transmission power too much even in the case of no interference.

Earlier studies have shown that the selfish nature of the network users and because of the lack of information about other users actions, the system converges to an equilibrium where the best choice is to distribute power *P* in all the available channels [10, 12]. Clearly that is not the most favorable situation for the overall network, because of the unnecessary interference and the high battery drain. Even though a better condition is reached when every user chooses a different channel, the results are not good for cooperative users if there is a group of users that do selfishly. Hereinafter, we define the main aspects of the classic problem, after that, we will explain the necessary changes to build the quantum model and the advantages that it brings with.

In this model, as in the real scenario, *N* independent network users with no information about other user actions are considered. They are free to choose among *N* channels, so they can use only one (cooperative attitude), otherwise they can distribute their power among all the available channels (selfish attitude). It is supposed that every user may transmit a total power *P*, that is, in one channel or distributed through all the channels. The payoffs obtained by the players are linked to the Signal-to-interference-and-noise-ratio (SINR), that is, the greater the SINR the better is the reception quality. The signal *S* is related to the power used by one player, the interference *I* is related to the power received from the other players. As a consequence, the best way to assign value to the reward (payoff) some player *j* obtains from the network is through the Shannon Capacity, [15], given by equation (1), where $\alpha_j^k P$ is the power that transmitter *j* assigns to channel *k* and $h_{ij}(k)$ is the *k* channel gain between the transmitter j^{th} and receiver i^{th}. Moreover, $\sigma(k)$ is the thermal noise at the receiver on channel *k.*

$$C_j = \sum_{k=1}^{N} \log_2 \left(1 + \frac{h_{ij}(k).\alpha_j^k P}{\sigma(k) + \sum_{i \neq j}^{N} \alpha_j^k P.h_{ij}(k)} \right) \quad (1)$$

The players strategies consist on choosing the fractions of *P* assigned to each channel by means of $A_j = [\alpha_j^1, \alpha_j^2, \ldots \alpha_j^N]$, thus $\alpha_j^i P$ denotes, for instance, the portion of power player *j* assigns to channel *i*. Thereby, the payoff each player receives will depend on the quality of the channel and the chosen action vector *A*.

In our model, the players must decide between two extreme options, these are Cooperating, which implies to select only one channel, or Defecting, which implies to distribute the power among all the channels. It is known that this problem has an inefficient solution classically, since the users finds to Defect as the best option. However, we show that it is possible to quantize the model in order to improve the players utilities. Consequently, in what follows we describe the characteristics of the quantum model.

$$|S0\rangle = \frac{|00\ldots0\rangle + i|11\ldots1\rangle}{\sqrt{2}} \quad (2)$$

In the first place, the system initial state $|S0\rangle$ depicted in (2) corresponds to the quantum superposition of the every user cooperating state, $|00\ldots0\rangle$ and the every user defecting state, $|11\ldots1\rangle$. So, note that "0" in some position *j* means that user $0 \le j \le N-1$ decides to cooperate and a "1" means the opposite situation. Note also that without the users intervention, the system outcome can only be one of this two situations with probability 1/2. Every user is aware about the initial state and they are able to transform the system final state according to their strategies. The users strategies must transform the system state in order to change the probability amplitudes of the corresponding states. Thus, the quantum strategies are represented by unitary operators on a Hilbert space (3) that the users apply locally to their qubit on the entangled state and transform the whole system behavior accordingly.

$$U_i(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & e^{i\phi}\sin\frac{\theta}{2} \\ -e^{-i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad (3)$$

The selection of some $0 < \theta < \pi$ and $0 < \phi < \frac{\pi}{2}$ implies covering linear combinations of strategies whose application drops outcomes that are not different from the classic game with mixed strategies and other combinations of (θ,ϕ) which lead to outcomes that are not possible in the classic game and consequently new equilibrium points emerge [16].

$$|H(0)|^2 = |H(1)|^2 = |H(2)|^2 = \begin{bmatrix} 1 & h & h \\ h & 1 & h \\ h & h & 1 \end{bmatrix} \qquad (4)$$

In order to clarify some concepts we present the case of *N*=3 players transmitting over the same number of channels. Besides, for the sake of simplicity and without affecting the problem generality, we consider the normalization $h_{ij}(k)$ hen *i*=*j* and $h_{ij}(k) = h$ for any other case, being (4) the network matrix. Suppose *A, B* and *C* denote three players whose utilities are calculated using expression (1), that is, the classical case. For instance, the player *A* utilities are displayed in Table **1** as function of his and the other users actions. As the table shows, the highest utility results when he defects and the other players cooperate $|ABC\rangle = 7.65$. On the other hand, if the other players betray while *A* cooperates, he receives a significantly smaller payoff. Because of that, classically all players decide to betray on average and resulting 2.42 the channel capacity for any user. In other words, if, for example, it is supposed the minimum Capacity C_j admitted is 2, the players will prefer a clear communication at the expense of the battery drain. This clearly is the Nash Equilibrium for the network and the corresponding payoff results less than the one the players would achieve if they all cooperate. This situation is shown in Figure **1** where the user *C* payoff is depicted as function of others users actions. The upper surface corresponds to the case of user *C* deciding to Betray and the lower surface graph arises from the *C* decision to Cooperate.

Table 1: Player *A* capacity for *SNR*=100 and *h*=0.23. 0=*Cooperate* and 1=*Defect*. The highest utility results when *A* defects and the others cooperate. On the other hand, if the others betray while *A* cooperates, the last receive significantly smaller payoff. Because of the symmetry, the same occurs with other players payoffs.

$\lvert ABC\rangle$	C_A
$\lvert 000\rangle$	3.3291
$\lvert 001\rangle$	1.8339
$\lvert 010\rangle$	1.8339
$\lvert 011\rangle$	1.425
$\lvert 100\rangle$	7.65
$\lvert 101\rangle$	3.44
$\lvert 110\rangle$	3.44
$\lvert 111\rangle$	2.42

When the problem is analyzed from the point of view of quantum computation, that is, when the initial state is entangled and the strategy space is spanned to add new strategies, it is possible to present new favorable conditions to the users. In other words, they can make use of some strategy which leads to a more favorable situation for the entire network.

Preparing the system in the entangled state (2) defined above; the players choose their strategies according to their preferences and their previous experience. Meanwhile, the classic strategy "Cooperate" is represented by $U(0,0)$ while betraying is represented by strategy $U(\pi\frac{\pi}{2})$, and, as Figure **2** shows, the Player *C* payoff depends on the (θ,ϕ) combinations. The data of *C* payoff depicted in the figure arise when the strategies of *A* and *B* are $Q = U(\pi,0)$. As a consequence, it is observed that player *C* best strategy is also to choose $U(\pi,0)$ and because of the problem symmetry, *Q* results the best strategy for every player. Moreover, this is a Pareto optimum due to none user is willing to change because the payoff would be less than 3.3291, the one corresponding to the strategy, $|000\rangle$.

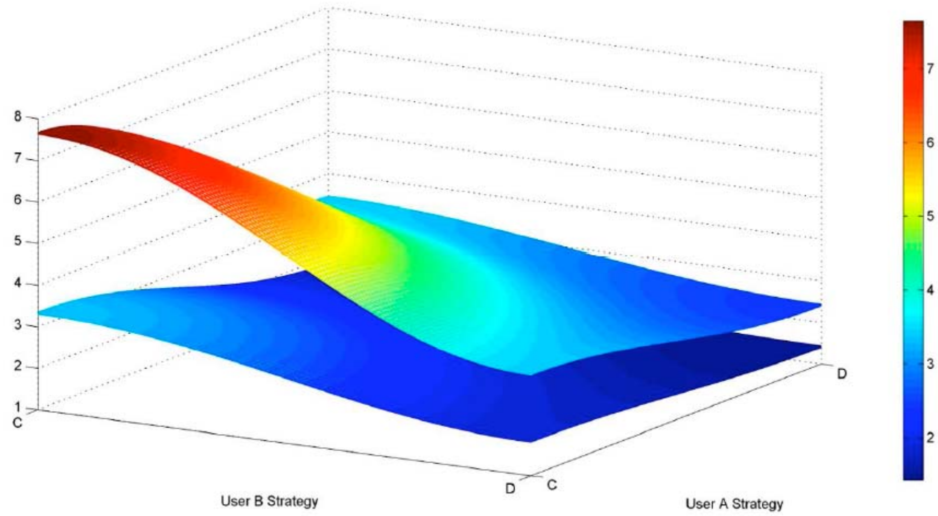

Figure 1: Player's *C* payoff as function of θ and ϕ. Users *A* and *B* play both strategy $Q = U(\pi,0)$. The maximum payoff is clearly $(\pi,0)$ and then, the best strategy for *C* is to play also $U(\pi,0)$.

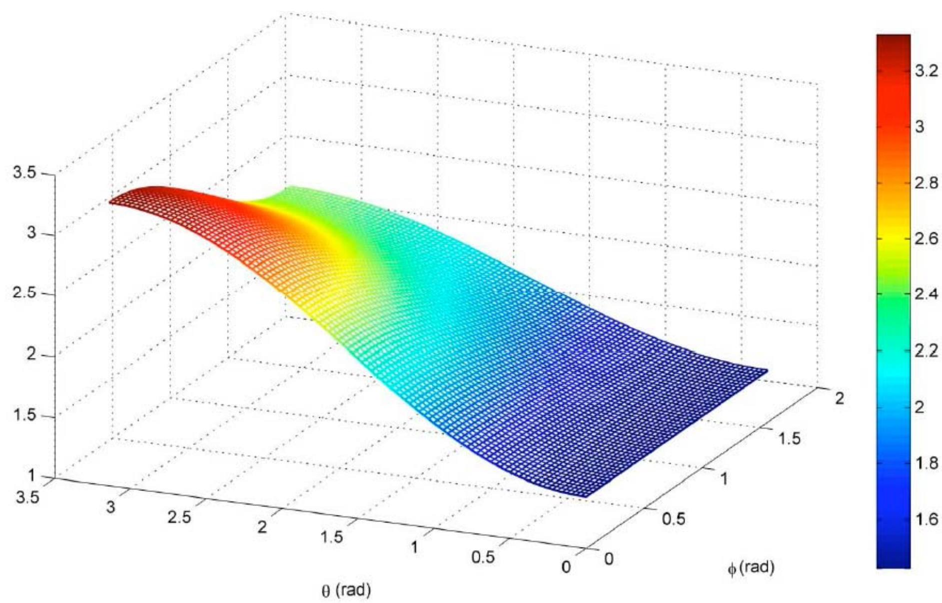

Figure 2: Player C payoff as function of θ and ϕ Users *A* and *B* play both strategy $Q = U(\pi,0)$. The maximum payoff is clearly $(\pi,0)$ and then, the best strategy for *C* is to play also $U(\pi,0)$.

Consequently, the quantum model offers the users a different way of stable cooperation, allowing better transmissions and less battery drain.

3. CONCLUSIONS

We have proposed a novel quantum game application. The selfish behavior of users in a wireless network can be naturally modeled by a game. The nodes of the network are the players and the payoffs are represented by the users transmission rate. The classic strategies are Cooperate with the whole network, which implies to direct all the power to one channel, or to betray, distributing the power to all the channels causing interference to other users and diminishing their SINR. The quantum game of interference can describe classic users behavior but also permits to design new power control techniques for improving the actual ones. Cooperating is the best choice because of power saving and interference avoiding but is not an equilibrium condition, since any player can do better if change the strategy to "Betray". On the other hand, the use of quantum entanglement make possible the use of a different strategy which drives to a Pareto optimal equilibrium, guaranteeing the best possible performance for the whole network in the sense of better transmission rate and more power saving.

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REFERENCES

[1] Arizmendi CM, Barrangu JP, Zabaleta OG. A 802.11 MAC protocol adaptation for quantum communications. In Distributed Simulation and Real Time Applications (DS-RT), 2012 IEEE/ACM 16th International Symposium on, Oct. 2012; pp. 147-150.

[2] Zabaleta OG, Arizmendi CM. Quantum dating market. Physica A 2010; 389: 2858-2863. http://dx.doi.org/10.1016/j.physa.2010.03.010

[3] Abbas MM, Mahmood H. Advances in QUANTUM MECHANICS, Chapter Name: Power Control in Ad Hoc Networks. Intech, China 2011.

[4] Xiao Y, Shan X, Ren Y. Game theory models for IEEE 802.11 DCF in wireless ad hoc networks. Communications Magazine IEEE 2005; 43: 22-26. http://dx.doi.org/10.1109/MCOM.2005.1404594

[5] Arizmendi CM. Paradoxical way for losers in a dating game. In Osvaldo A. Rosso Orazio Descalzi and Hilda A. Larrondo, editors, Proc. AIP Nonequilibrium Statistical Mechanics and Nonlinear Physics, Mar del Plata, Argentina, December 2006; pp. 20-25.

[6] Meyer DA. Quantum strategies. Phys Rev Lett 1999; 82: 1052-1055. http://dx.doi.org/10.1103/PhysRevLett.82.1052

[7] Romanelli A. Quantum games *via* search algorithms. Physica A 2007; 379: 545-551. http://dx.doi.org/10.1016/j.physa.2007.02.029

[8] Schmidt AGM, Da Silva L. Quantum russian roulette. Physica A: Statistical Mechanics and its Applications 2013; 392(2): 400-410.

[9] Arizmendi CM, Zabaleta OG. Stability of couples in a quantum dating market. Special IJAMAS issue: Statistical Chaos and Complexity 2012; 26: 143-149.

[10] Laufer A, Leshem A. Distributed coordination of spectrum and the prisoner's dilemma. In New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. First IEEE International Symposium on, Nov. 2005; pp. 94-100.

[11] Yu W, Rhee W, Boyd S, Ciofli, JM. Iterative water-filling for gaussian vector multiple access channels. In Information Theory, 2001. Proceedings. IEEE International Symposium on, pp. 322.

[12] Yu W, Ginis G, Cioffi JM. Distributed multiuser power control for digital subscriber lines. IEEE Journal on Selected Areas in Communications 2002; 20(5): 1105-1115. http://dx.doi.org/10.1109/JSAC.2002.1007390

[13] Axelrod R, Hamilton W. The evolution of cooperation. Science 1981; 211: 1390-1396. http://dx.doi.org/10.1126/science.7466396

[14] Eisert J, Wilkens M, Lewenstein M. Quantum games and quantum strategies. Phys Rev Lett 1999; 83: 3077-3080. http://dx.doi.org/10.1103/PhysRevLett.83.3077

[15] William Stallings. Wireless Communications and Networks. Pearson Prentice Hall, Upper Saddle River, NJ, 2nd. edition, 2002.

[16] Du J, Xu X, Hui L, Zhou X, Han R. Playing prisoner's dilemma with quantum rules. Fluctuation and Noise Letters, 2002; 2(4): 189-R203. http://dx.doi.org/10.1142/S0219477502000993

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